15

A historical journey through the internal stabilization problem

Alban Quadrat
INRIA Sophia Antipolis,
APICS project,
2004 Route des Lucioles, BP 93,
06902 Sophia Antipolis Cedex,
France.
Alban.Quadrat@sophia.inria.fr

Abstract

The purpose of this talk is to give a historical but personal journey through the internal stabilization problem. We study the evolution of the mathematical formulation of this concept and its characterizations from the seventies to the present day. In particular, we explain how the different mathematical formulations allow one to parametrize all the stabilizing controllers of an internally stabilizable plant. Finally, we focus on the important contributions of F. M. Callier on the internal stabilization problem of classes of infinite-dimensional systems.

Keywords

Internal stabilization problem, parametrization of all stabilizing controllers, doubly coprime factorizations, infinite-dimensional linear systems, fractional representation approach, fractional ideals, lattices, algebraic analysis.

Recognizing when a real plant can be stabilized by means of a feedback law is one of the oldest issues in automatic control. This problem, developed for clear practical reasons, was recently abstracted within the mathematical language in order to be studied on its own and generalized to larger and larger classes of systems, slowly passing from the engineer world to the mathematical one. With a very few concepts such as controllability, observability and robustness, the concept of stabilizability is one of the main interesting cross-fertilizations between very practical engineering problems and mathematics. The evolution of this new mathematical concept should attract more attention from science historians and researchers as we shall show.

We want to take the opportunity of the celebration of F. M. Callier's scientific career who, with C. A. Desoer, G. Zames, M. Vidyasagar, B. A. Francis and others, has brought significant contributions to the study of this concept particularly for infinite-dimensional linear systems ([3, 4, 5, 7, 9, 21, 26]), to give a historical but personal journey through the internal stabilization problem. We are convinced that there is a lot to learn from the historical study of this central concept. Reading directly the papers where this concept was created, developed and used (see, e.g., [8, 10, 13, 16, 22, 27] and the references therein) is a source of enlightenment, bringing a new light on the evolutions developed since and the comings and goings between different approaches. See [2, 24] for some historical accounts.

We study the evolution of the mathematical formulation of the concept of internal stabilizability and its characterizations from the seventies to the present day. We explain how the different mathematical formulations allowed one to parametrize all the stabilizing controllers of the corresponding plant. We emphasize on the fractional representation approach developed by M. Vidyasagar, C. A. Desoer, F. M. Callier, B. A. Francis and others based on the existence of doubly coprime factorizations of the transfer matrices ([6, 10, 15, 22, 23]) and on a mainly forgotten approach developed by G. Zames and B. A. Francis based on the particular transfer matrix $Q = C\,(I - P\,C)^{-1}$ ([13, 27]). See also [1, 2, 11, 12] for the second one. In particular, we focus on the significant contributions of F. M. Callier on the internal stabilization problem of infinite-dimensional linear systems (see, e.g., [3, 4, 5, 7]).

We explain how the use of modern algebraic techniques (fractional ideals, lattices, modules) allows us to show that the approach developed by G. Zames and B. A. Francis ([13, 27]) supersedes the classical fractional representation approach ([6, 10, 15, 22, 23]). Within this lattice approach ([18, 19]), we give general necessary and sufficient conditions for internal stabilizability and for the existence of (weakly) doubly coprime factorizations of irrational transfer matrices. Moreover, we give a general parametrization of all stabilizing controllers of an internally stabilizable plant which reduces to the classical Youla-Kučera parametrization ([10, 14, 25]) when the plant admits a doubly coprime factorization ([18, 20]). The knowledge of only one stabilizing controller is required to get this new parametrization.

Finally, we explain why the lattice approach was historically developed in algebra by Kummer, Dedekind and their followers at the end of the nineteen century for solving conditions similar to the ones obtained from the characterization of internal stabilizability (and from Lamé's famous mistake on Fermat's last theorem). Hence, the use of this mathematical theory was very natural and allowed us to develop our results before realizing that the main ideas could be traced back to the pioneering work of G. Zames and B. A. Francis ([13, 27]). These ideas could not have been completely realized for general classes of systems as the authors did not know the fractional ideal and lattice approaches. Therefore, this shows that old approaches can sometimes be still fruitful when the corresponding mathematical techniques are mature even if, as it was unfortunately our case, we had to preliminary rediscover them before investigating the past literature! The moral of this story advocates for the better knowledge of the historical development of our field and explains the topic of this talk, hoping closing the loop!

Bibliography

[1] A. Bhaya, C. A. Desoer. Necessary and sufficient conditions on $Q\,(= C\,(I + P\,C)^{-1})$ for stabilization of the linear feedback system $S(P, C)$. *Systems and Control Letters*, 7, 35-38, 1986.

[2] S. Boyd, C. Barratt, S. Norman. Linear controller design: limits of performance via convex optimization. *Proceedings of the IEEE*, 78 (3), 529-574, 1990.

[3] F. M. Callier, C. A. Desoer. An algebra of transfer functions for distributed linear time-invariant systems. *IEEE Trans. Circuits Systems*, 25 (9), 651-662, 1978.

[4] F. M. Callier, C. A. Desoer. Simplification and new connections on an algebra of transfer functions for distributed linear time-invariant systems. *IEEE Trans. Circuits Systems*, 27 (4), 320-323, 1980.

[5] F. M. Callier, C. A. Desoer. Stabilization, tracking and disturbance rejection in multivariable convolution systems. *Annales de la Société Scientifique de Bruxelles*, 94 (I), 7-51, 1980.

[6] R. Curtain, H. J. Zwart. *An Introduction to Infinite-Dimensional Linear Systems Theory*. Texts in Applied Mathematics 21, Springer-Verlag, 1995.

[7] C. A. Desoer, F. Callier. Convolution feedback systems. *SIAM J. Control*, 10 (4), 736-746, 1972.

[8] C. A. Desoer, W. S. Chan. The feedback interconnection of lumped linear time-invariant systems. *J. Franklin Inst.*, 300 (5-6), 325-351, 1975.

[9] C. A. Desoer, M. Vidysagar. *Feedback Systems: Input-Output Properties*. Academic Press, 1975.

[10] C. A. Desoer, R.-W. Liu, J. Murray, R. Saeks. Feedback system design: the fractional representation approach to analysis and synthesis. *IEEE Trans. Automat. Contr.*, 25 (3), 399-412, 1980.

[11] C. A. Desoer, M. J. Chen. Design of Multivariable feedback systems with stable plants. *IEEE Trans. Automat. Contr.*, 26 (2), 408-415, 1981.

[12] C. A. Desoer, C. L. Gustafson. Design of multivariable feedback systems with simple unstable plant. *IEEE Trans. Automat. Contr.*, 29 (10), 901-908, 1984.

[13] B. A. Francis, G. Zames. On H^∞-optimal sensitivity theory for SISO feedback systems. *IEEE Trans. Automat. Contr.*, 29 (1), 9-16, 1984.

[14] V. Kučera. *Discrete Linear Control: The Polynomial Equation Approach*. Wiley, 1979.

[15] H. Logemann. Stabilization and regulation of infinite-dimensional systems using coprime factorizations. in Lecture Notes in Control and Information Sciences 185, Analysis and Optimization of Systems: State and Frequency Domain Approaches for Infinite-Dimensional Systems, R. Curtain ed., 103-139, 1993.

[16] G. C. Newton, L. A. Gould, J. Kaiser. *Design of Linear Feedback Control*. Wiley, 1957.

[17] A. Quadrat. The fractional representation approach to synthesis problems: Part I: (Weakly) doubly coprime factorizations, Part II: Internal stabilization. *SIAM J. Control Optimization*, 42 (1), 266-299, 300-320.

[18] A. Quadrat. On a generalization of the Youla-Kučera parametrization. Part I: The fractional ideal approach to SISO system. *Systems and Control Letters*, 50 (2), 135-148, 2003.

[19] A. Quadrat. A lattice approach to analysis and synthesis problems. *Mathematics of Control, Signals, and Systems*, 18 (2), 147-186, 2006.

[20] A. Quadrat. On a generalization of the Youla-Kučera parametrization. Part II: The lattice approach to MIMO systems. *Mathematics of Control, Signals, and Systems*, 18 (3), 199-235.

[21] M. Vidyasagar. Input-output stability of a broad class of linear time-invariant multivariable systems. *SIAM J. Control*, 10 (1), 203-209, 1972.

[22] M. Vidyasagar, H. Schneider, B. A. Francis. Algebraic and topological aspects of feedback stabilization. *IEEE Trans. Automat. Contr.*, 27 (4), 880-, 894,1982.

[23] M. Vidyasagar. *Control System Synthesis: A Factorization Approach*. The MIT Press, 1985.

[24] M. Vidyasagar. A brief history of the graph topology. *European Journal of Control*, 2, 80-87, 1996.

[25] D. C. Youla, H. A. Jabr, J. J. Bongiorno. Modern Wiener-Hopf design of optimal controllers. Part II: The multivariable case. *IEEE Trans. Automat. Contr.*, 21 (3), 319-338, 1976.

[26] G. Zames. Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms, and approximate inverses. *IEEE Trans. Automat. Contr.*, 26 (2), 301-320, 1981.

[27] G. Zames, B. A. Francis. Feedback, minimax sensitivity, and optimal robustness. *IEEE Trans. Automat. Contr.*, 28 (5), 585-601, 1983.