Domain-Adversarial Neural Networks

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Outline

Our Domain Adaptation Setting

Binary classification tasks

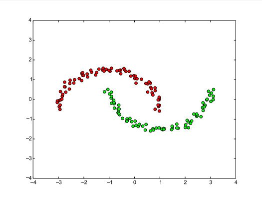
- Input space: $\mathbb{R}^d$
- Labels: $\{0, 1\}$

Two different data distributions

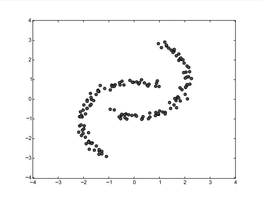
- Source domain: $\mathcal{D}_S$
- Target domain: $\mathcal{D}_T$

A **domain adaptation** learning algorithm is provided with

a **labeled source sample**
$S = \{(\mathbf{x}_i^s, y_i^s)\}_{i=1}^m \sim (\mathcal{D}_S)^m$,



an **unlabeled target sample**
$T = \{\mathbf{x}_i^t\}_{i=1}^m \sim (\mathcal{D}_T)^m$.



The goal is to build a classifier $\eta : \mathbb{R}^d \to \{0,1\}$ with a low **target risk**

$$R_{\mathcal{D}_T}(\eta) \stackrel{\text{def}}{=} \Pr_{(\mathbf{x}^t, y^t) \sim \mathcal{D}_T} \left[\eta(\mathbf{x}^t) \neq y^t \right].$$

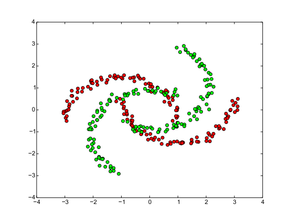
Divergence between source and target domains

Definition (Ben David et al., 2006)

Given two domain distributions $\mathcal{D}_S$ and $\mathcal{D}_T$, and a **hypothesis class** $\mathcal{H}$, the $\mathcal{H}$-**divergence** between $\mathcal{D}_S$ and $\mathcal{D}_T$ is

$$
\begin{aligned}
d_{\mathcal{H}}(\mathcal{D}_S, \mathcal{D}_T) &\stackrel{\text{def}}{=} 2 \sup_{\eta \in \mathcal{H}} \left| \Pr_{\mathbf{x}^s \sim \mathcal{D}_S} \left[\eta(\mathbf{x}^s) = 1 \right] - \Pr_{\mathbf{x}^t \sim \mathcal{D}_T} \left[\eta(\mathbf{x}^t) = 1 \right] \right| . \\
&= 2 \sup_{\eta \in \mathcal{H}} \left| \Pr_{\mathbf{x}^s \sim \mathcal{D}_S} \left[\eta(\mathbf{x}^s) = 1 \right] + \Pr_{\mathbf{x}^t \sim \mathcal{D}_T} \left[\eta(\mathbf{x}^t) = 0 \right] - 1 \right| .
\end{aligned}
$$

The $\mathcal{H}$-**divergence** measures the ability of an hypothesis class $\mathcal{H}$ to **discriminate** between source $\mathcal{D}_S$ and target $\mathcal{D}_T$ distributions.



Bound on the target risk

Theorem (Ben David et al., 2006)

Let $\mathcal{H}$ be a hypothesis class of VC-dimension d. With probability $1-\delta$ over the choice of samples $S \sim (\mathcal{D}_S)^m$ and $T \sim (\mathcal{D}_T)^m$, for every $\eta \in \mathcal{H}$:

$$R_{\mathcal{D}_T}(\eta) \;\leq\; R_S(\eta) + \frac{4}{m}\sqrt{d \log \tfrac{2e\,m}{d} + \log \tfrac{4}{\delta}} + \hat{d}_{\mathcal{H}}(S,T) + \frac{4}{m^2}\sqrt{d \log \tfrac{2\,m}{d} + \log \tfrac{4}{\delta}} + \beta$$

with $\beta \geq \inf_{\eta^* \in \mathcal{H}} \left[R_{\mathcal{D}_S}(\eta^*) + R_{\mathcal{D}_T}(\eta^*) \right]$.

Empirical risk on the source sample:

$$R_S(\eta) \;\stackrel{\text{def}}{=}\; \frac{1}{m}\sum_{i=1}^{m} I[\eta(\mathbf{x}_i^s) \neq y_i^s]\,.$$

Empirical $\mathcal{H}$-divergence:

$$\hat{d}_{\mathcal{H}}(S,T) \;\stackrel{\text{def}}{=}\; 2\max_{\eta \in \mathcal{H}} \left[\frac{1}{m}\sum_{i=1}^{m} I[\eta(\mathbf{x}_i^s) = 1] + \frac{1}{m}\sum_{i=1}^{m} I[\eta(\mathbf{x}_i^t) = 0] - 1 \right].$$

Bound on the target risk

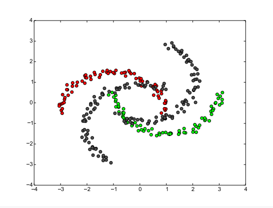
Theorem (Ben David et al., 2006)

Let $\mathcal{H}$ be a hypothesis class of VC-dimension d. With probability $1-\delta$ over the choice of samples $S \sim (\mathcal{D}_S)^m$ and $T \sim (\mathcal{D}_T)^m$, for every $\eta \in \mathcal{H}$:

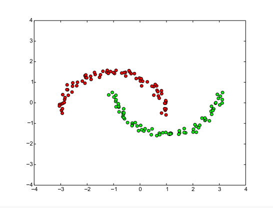
$$R_{\mathcal{D}_T}(\eta) \;\leq\; R_S(\eta) + \frac{4}{m}\sqrt{d \log \frac{2e\,m}{d} + \log \frac{4}{\delta}} + \hat{d}_{\mathcal{H}}(S,T) + \frac{4}{m^2}\sqrt{d \log \frac{2\,m}{d} + \log \frac{4}{\delta}} + \beta$$

with $\beta \geq \inf_{\eta^* \in \mathcal{H}} \left[R_{\mathcal{D}_S}(\eta^*) + R_{\mathcal{D}_T}(\eta^*) \right]$.

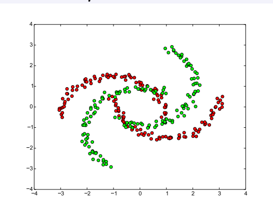
Target risk $R_{\mathcal{D}_T}(\eta)$ is low
if, given S and T,



$R_S(\eta)$ is small,
i.e., $\eta \in \mathcal{H}$ is good on



and $\hat{d}_{\mathcal{H}}(S,T)$ is small,
i.e., all $\eta' \in \mathcal{H}$ are bad on

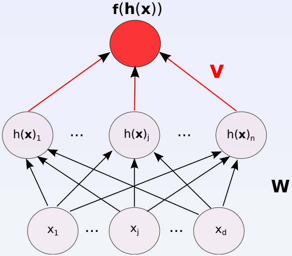


Standard Neural Network

Let consider a neural network architecture with one hidden layer

$$\mathbf{h}(\mathbf{x}) = \mathrm{sigm}(\mathbf{b} + \mathbf{W}\mathbf{x}), \quad \text{and} \quad \mathbf{f}(\mathbf{h}(\mathbf{x})) = \mathrm{softmax}(\mathbf{c} + \mathbf{V}\mathbf{h}(\mathbf{x})).$$

$$\min_{\mathbf{W},\mathbf{V},\mathbf{b},\mathbf{c}} \underbrace{\left[\frac{1}{m}\sum_{i=1}^{m} -\log\left(1 - y_i^s - \mathbf{f}(\mathbf{h}(\mathbf{x}_i^s))\right)\right]}_{\text{source loss}}.$$



Given a source sample $S = \{(\mathbf{x}_i^s, y_i^s)\}_{i=1}^m \sim (\mathcal{D}_S)^m$,

1. Pick a $\mathbf{x}^s \in S$
2. Update $\mathbf{V}$ towards $\mathbf{f}(\mathbf{h}(\mathbf{x}^s)) = y^s$
3. Update $\mathbf{W}$ towards $\mathbf{f}(\mathbf{h}(\mathbf{x}^s)) = y^s$

The hidden layer learns a **representation $\mathbf{h}(\cdot)$** from which linear hypothesis $\mathbf{f}(\cdot)$ can **classify source examples**.

Domain-Adversarial Neural Network (DANN)

Empirical $\mathcal{H}$-divergence

$$\hat{d}_{\mathcal{H}}(S, T) \stackrel{\text{def}}{=} 2 \max_{\eta \in \mathcal{H}} \left[\frac{1}{m} \sum_{i=1}^{m} I[\eta(\mathbf{x}_i^s) = 1] + \frac{1}{m} \sum_{i=1}^{m} I[\eta(\mathbf{x}_i^t) = 0] - 1 \right].$$

We estimate the $\mathcal{H}$-divergence by a logistic regressor that model the probability that a given input (either $\mathbf{x}^s$ or $\mathbf{x}^t$) is from the source domain:

$$o(\mathbf{h}(\mathbf{x})) \stackrel{\text{def}}{=} \operatorname{sigm}(d + \mathbf{w}^\top \mathbf{h}(\mathbf{x})).$$

Given a representation output by the hidden layer $\mathbf{h}(\cdot)$:

$$\hat{d}_{\mathcal{H}}\Big(\mathbf{h}(S), \mathbf{h}(T)\Big) \approx 2 \max_{\mathbf{w}, d} \left[\frac{1}{m} \sum_{i=1}^{m} \log\big(o(\mathbf{h}(\mathbf{x}_i^s))\big) + \frac{1}{m} \sum_{i=1}^{m} \log\big(1 - o(\mathbf{h}(\mathbf{x}_i^t))\big) - 1 \right].$$

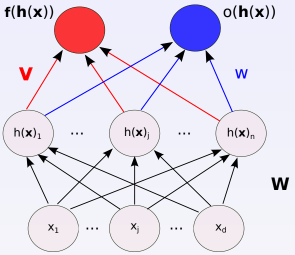
Domain-Adversarial Neural Network (DANN)

$$\min_{\mathbf{W},\mathbf{V},\mathbf{b},\mathbf{c}} \Bigg[\underbrace{\frac{1}{m}\sum_{i=1}^{m} -\log\big(1 - y_i^s - \mathbf{f}(\mathbf{h}(\mathbf{x}_i^s))\big)}_{\text{source loss}} + \underbrace{\lambda \max_{\mathbf{w},d} \left(\frac{1}{m}\sum_{i=1}^{m} \log\big(o(\mathbf{h}(\mathbf{x}_i^s))\big) + \frac{1}{m}\sum_{i=1}^{m} \log\big(1 - o(\mathbf{h}(\mathbf{x}_i^t))\big) \right)}_{\text{adaptation regularizer}} \Bigg],$$

where $\lambda > 0$ weights the domain adaptation regularization term.

Given a source sample $S = \{(\mathbf{x}_i^s, y_i^s)\}_{i=1}^m \sim (\mathcal{D}_S)^m$,
and a target sample $T = \{(\mathbf{x}_i^t)\}_{i=1}^m \sim (\mathcal{D}_T)^m$,

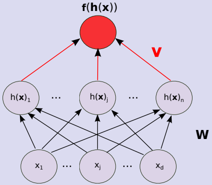
1. Pick a $\mathbf{x}^s \in S$ and $\mathbf{x}^t \in T$
2. Update $\mathbf{V}$ towards $\mathbf{f}(\mathbf{h}(\mathbf{x}^s)) = y^s$
3. Update $\mathbf{W}$ towards $\mathbf{f}(\mathbf{h}(\mathbf{x}^s)) = y^s$
4. Update $\mathbf{w}$ towards $o(\mathbf{h}(\mathbf{x}^s)) = 1$ and $o(\mathbf{h}(\mathbf{x}^t)) = 0$
5. Update $\mathbf{W}$ towards $o(\mathbf{h}(\mathbf{x}^s)) = 0$ and $o(\mathbf{h}(\mathbf{x}^t)) = 1$



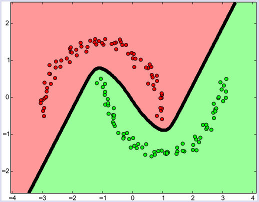
DANN finds a representation h(·) that are good on S;
but unable to discriminate between S and T.

Toy Dataset

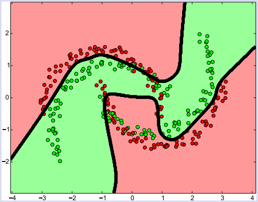
Standard Neural Network (NN)



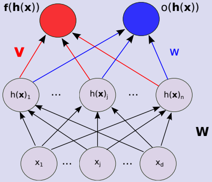
Trained to classify source



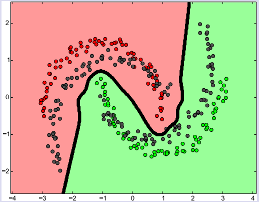
Trained to classify domains



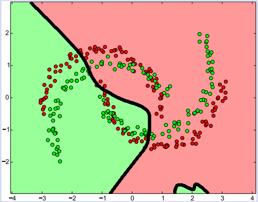
Domain-Adversarial Neural Networks (DANN)



Classification output: $\mathbf{f}(\mathbf{h}(\mathbf{x}))$



Domain output: $o(\mathbf{h}(\mathbf{x}))$



Amazon Reviews

Input: product review (bag of words) — **Output:** positive or negative rating.

Dataset	DANN	NN
books $\rightarrow$ dvd	0.201	**0.199**
books $\rightarrow$ electronics	**0.246**	0.251
books $\rightarrow$ kitchen	**0.230**	0.235
dvd $\rightarrow$ books	**0.247**	0.261
dvd $\rightarrow$ electronics	**0.247**	0.256
dvd $\rightarrow$ kitchen	0.227	0.227
electronics $\rightarrow$ books	**0.280**	0.281
electronics $\rightarrow$ dvd	**0.273**	0.277
electronics $\rightarrow$ kitchen	**0.148**	0.149
kitchen $\rightarrow$ books	**0.283**	0.288
kitchen $\rightarrow$ dvd	0.261	0.261
kitchen $\rightarrow$ electronics	0.161	0.161

Note: We use a *small labeled subset* of 100 target examples to select the hyperparameters.

Marginalized Stacked Denoising Autoencoders (mSDA)

Question

Does DANN can be combined with other representation learning techniques for domain adaptation?

The autoencoders mSDA (Chen et al. 2012) provides a new common representation for source and target (unsupervised)

With **mSDA+SVM**, Chen et al. (2012) obtained *state-of-the-art* results on Amazon Reviews:
– Train a linear SVM on mSDA source representations.

We try **mSDA+DANN**:
– Train DANN on source representations and target representations.

Amazon Reviews

Input: product review (bag of words) — **Output:** positive or negative rating.

Dataset	**mSDA+DANN**	**mSDA+SVM**
books $\rightarrow$ dvd	0.176	**0.175**
books $\rightarrow$ electronics	**0.197**	0.244
books $\rightarrow$ kitchen	**0.169**	0.172
dvd $\rightarrow$ books	0.176	0.176
dvd $\rightarrow$ electronics	**0.181**	0.220
dvd $\rightarrow$ kitchen	**0.151**	0.178
electronics $\rightarrow$ books	0.237	**0.229**
electronics $\rightarrow$ dvd	**0.216**	0.261
electronics $\rightarrow$ kitchen	**0.118**	0.137
kitchen $\rightarrow$ books	**0.222**	0.234
kitchen $\rightarrow$ dvd	**0.208**	0.209
kitchen $\rightarrow$ electronics	0.141	**0.138**

Note: We use a *small labeled subset* of 100 target examples to select the hyperparameters. The *noise parameter* of mSDA representations is fixed to 50%.

Future Work

Several paths to explore:

- Deeper neural networks architectures.
- Multiclass / Multilabels problems.
- Multisource domain adaptation.

Thank you!