ONE YEAR POSTDOCTORAL FELLOWSHIP

ANDRÉ DE LAIRE, GUILLAUME DUJARDIN, INGRID LACROIX-VIOLET

The purpose of this research project is to study different forms of the Landau-Lifshitz equation that constitutes a fundamental equation in the magnetic recording industry [12]. The evolution in time for the magnetization $\vec{m} : \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^3$ is given by the Landau-Lifshitz (LL) equation

$$\partial_t \vec{m} + \vec{m} \times H_{\text{eff}}(\vec{m}) = 0, \quad \|\vec{m}\| = 1.$$

The effective field $H_{\text{eff}}(\vec{m})$ is determined by the characteristics of the material (exchange energy, anisotropy, magnetostatic interactions, etc) [9]. This equation describes a precession of the magnetization around the effective field. A related equation is the Landau-Lifshitz-Gilbert (LLG) equation that includes a dissipative term to take into account some possible damping effects due to some intrinsic loss in the material.

As special cases of these equations, we have the heat-flow for harmonic maps and the Schrödinger map equation onto the sphere $\mathbb{S}^2$. Using some changes of variables, the LLG equation can be seen as a nonlocal cubic Schrödinger equation as well as a cubic quasilinear Schrödinger equation [8].

The LL equation is highly nonlinear due to the cross product in the equation and the constraint $\|\vec{m}\| = 1$ and it provides a source of rich nonlinear phenomena. Depending on the nature of the interactions, this equation exhibits several types on nonlinear structures (solitons, breathers, spin waves).

In the physical literature there are formal arguments showing that, depending on the types of the materials, the LL equation can be approximated by the cubic NLS or by the Sine-Gordon equation [7]. In a recent paper A. de Laire and P. Gravejat have proved a result in this direction [5] and they have given estimates of the difference in Sobolev norms between a solution of the LL equation and a solution of the Sine-Gordon equation. These differences are small in biaxial ferromagnetics with a strong easy-plane anisotropy and the proofs are valid until a certain time. This result holds in any dimension. It will be interesting to perform some numerical simulations to see if these estimates and the time of validity are sharp or if it is possible to improve them. The same questions arise in the NLS approximation of the LL equation. In this case there is still no rigorous work, and we propose to address this problem from both numerical and theoretical points of view.

Another possible branch of investigation is the study of solitons of the LL equation in dimension two. Using formal developments and numerical simulations, N. Papanicolaou and P. N. Spathis [11] found, in dimensions two and three, nonconstant, finite energy, localized, traveling waves, propagating with speed $c \in (0, 1)$. The existence of these solitons has been established rigorously in [10], but only for small speeds. In a recent work, D. Chiron and C. Scheid [6] have found numerically solitons with more energy and with more vortices in the context of the Gross-Pitaevskii equation. It is possible that these kinds of solitons also exist for the LL equation, and some numerical simulations could enlighten the situation.

The LLG equation has also close relationship with motion of a vortex filament (see [1]). In this context, the solutions have singularities and infinite energy. Therefore they do not lie

in classical Sobolev spaces and their rigorous study requires a careful choice of the functional framework. In [2, 3], A. de Laire, in collaboration with S. Gutierrez has shown the existence of *expander* self-similar solutions to the LLG equation and their stability in the BMO space. These solutions are related to the long time dynamics of the solutions of the LLG equation. As part of this research project, we propose to study theoretically and numerically the existence and stability of *shrinker* self-similar solutions, related to the formation of singularities and the existence of finite time blow-up.

References

[1] V. Banica and L. Vega, *Stability of the self-similar dynamics of a vortex filament*, Arch. Ration. Mech. Anal., 210(3):673–712, 2013.

[2] A. de Laire and S. Gutierrez, *Self-similar solutions and formation of singularities for the Landau-Lifshitz-Gilbert equation*, Nonlinearity, 28(5), 1307-1350, 2015.

[3] A. de Laire and S. Gutierrez, *The Cauchy problem for the Landau-Lifshitz-Gilbert equation in BMO and self-similar solutions.*, Nonlinearity, 28(5), 1307-1350, 2015.

[4] A. de Laire and P. Gravejat. *Stability in the energy space for chains of solitons of the Landau-Lifshitz equation*, arXiv:1701.03083.

[5] A. de Laire and P. Gravejat. *The Sine-Gordon regime of the Landau-Lifshitz equation with a strong easy-plane anisotropy* , arXiv:1705.08146.

[6] D. Chiron and C. Scheid. *Multiple branches of travelling waves for the Gross-Pitaevskii equation*, Preprint.

[7] L.D. Faddeev and L.A. Takhtajan. *Hamiltonian methods in the theory of solitons*, Classics in Mathematics. Springer-Verlag, Berlin, 2007.

[8] B. Guo and S. Ding, *Landau-Lifshitz equations*, volume 1 of Frontiers of Research with the Chinese Academy of Sciences. World Scientific, Hackensack, New Jersey, 2008.

[9] M. Lakshmanan, *The fascinating world of the Landau-Lifshitz-Gilbert equation: an overview*, Philos. Trans. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci., 369(1939):1280–1300, 2011.

[10] F. Lin and J. Wei. *Traveling wave solutions of the Schrödinger map equation*, Comm. Pure Appl. Math., 63(12):1585–1621, 2010.

[11] N. Papanicolaou and P. N. Spathis, *Semitopological solitons in planar ferromagnets*, Nonlinearity 12(2) 285–302, 1999.

[12] D. Wei, *Micromagnetics and Recording Materials*, SpringerBriefs in Applied Sciences and Technology, Springer-Verlag 2012.