Abstract

In this paper, we combine a descriptor approach to stability and control of linear systems with time-varying delays, which is based on the Lyapunov - Krasovskii techniques, with a recent result on sliding mode control of such systems. The systems under consideration have norm-bounded uncertainties and uncertain bounded delays. The solution is given in terms of linear matrix inequalities (LMIs) and improves the previous results based on other Lyapunov techniques. A numerical example illustrates the advantages of the new method.

1 Introduction

The interest in robust control of time-delay systems this last decade is witnessed by the rich dedicated literature (see for instance, [1]- [17] and the numerous references therein). Many existing results concern systems with unknown but constant delays. But in some applications, such as networked control or tele-operated systems, the assumption of a constant delay is too restrictive: this can lead to bad performances or, even worse, to unstability.

This paper combines two previous results so to obtain a more efficient sliding mode controller for uncertain systems with time-varying delays and norm-bounded uncertainties. Other results [9] concern varying delays but may lead to strong conditions which reduces the dynamic performances.

The first of these results is the sliding mode design given in [9], which copes with stabilization of systems with time-varying delays. The approach relies on the construction on a Lyapunov-Razumikhin function which allows fast variations of the delay but leads to some conservatism on the upper bound of the time-delay.
The second result given in [3] concerns the construction of a new class of Lyapunov-Krasovskii functionals using a descriptor model transformation. Unlike previous transformations, the descriptor model leads to a system which is equivalent to the original one (from the point of view of stability) and requires bounding of fewer cross-terms. Furthermore, following this approach, stability criteria have been given in [6] for systems with time-varying delays without any assumption on their derivatives (which was the case with the usual Lyapunov-Krasovskii functionals).

The paper is organized as follows: In section 2, we develop a Lyapunov-Krasovskii approach on a descriptor representation for an uncertain, linear, time-delay system. This provides a stability condition expressed in term of feasibility of a linear matrix inequality (LMI) (see [1]). Then, the design of a stabilizing memoryless state feedback is derived. Section 3 deals with the design of a sliding mode controller. This is achieved through the resolution of a generalized eigenvalue problem which can be solved efficiently using semi-definite programming tools. In the last section, an illustrative example is solved using our approach and comparison with previous results are provided.

**Notation:**
Throughout the paper the superscript ‘$T$’ stands for matrix transposition, $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space, $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices. The notation $P > 0$, for $P \in \mathbb{R}^{n \times n}$ means that $P$ is symmetric and positive definite. $I_n$ represents the $n \times n$ identity matrix.

## 2 Stabilization of linear systems with norm-bounded uncertainties by delayed feedback

In this section we consider the following uncertain linear system with a time-varying delay:

$$
\dot{x}(t) = (A_0 + H \Delta(t)E_0)x(t) + (A_1 + H \Delta(t)E_1)x(t - \tau(t)) + (B_0 + H \Delta(t)E_2)u(t) + B_1u(t - \tau(t)),
$$

$$
x(t) = \phi(t), \ t \in [-h, 0],
$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, $h$ is an upper-bound on the time-delay function ($0 \leq \tau(t) \leq h$, $\forall t \geq 0$). The matrix $\Delta(t) \in \mathbb{R}^{p \times q}$ is a matrix of time-varying, uncertain parameters satisfying

$$
\Delta^T(t)\Delta(t) \leq I_q \ \forall t.
$$

For simplicity, we consider only one delay, but the results of this section may be easily generalized to the case of multiple delays.

We seek a control law

$$
u(t) = Kx(t)
$$

that will asymptotically stabilize the system.
2.1 The stability issue

In this subsection, we consider the following equation:

\[ \dot{x}(t) = (\bar{A}_0 + H\Delta(t)\bar{E}_0)x(t) + (\bar{A}_1 + H\Delta(t)\bar{E}_1)x(t - \tau(t)). \]  

(4)

Representing (1) in an equivalent descriptor form [3]:

\[ \dot{x}(t) = y(t), \quad 0 = -y(t) + (\bar{A}_T + H\Delta\bar{E}_T)x(t) - (\bar{A}_1 + H\Delta\bar{E}_1)\int_{t-\tau(t)}^{t} y(s)ds \]

or

\[ E\dot{\bar{x}}(t) = \begin{bmatrix} 0 & I_n \\ \bar{A}_T + H\Delta\bar{E}_T & -I_n \end{bmatrix}\bar{x}(t) - \begin{bmatrix} 0 \\ \bar{A}_1 + H\Delta\bar{E}_1 \end{bmatrix}\int_{t-\tau(t)}^{t} y(s)ds, \]  

(5)

with

\[ \bar{x}(t) = \text{col}\{x(t), y(t)\}, \quad E = \text{diag}\{I_n, 0\}, \]

\[ \bar{A}_T = \bar{A}_0 + \bar{A}_1, \quad \bar{E}_T = \bar{E}_0 + \bar{E}_1, \]

the following Lyapunov-Krasovskii functional is applied:

\[ V(t) = \bar{x}^T(t)EP\bar{x}(t) + V_2(t), \]  

(6)

where

\[ P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad P_1 > 0, \quad EP = P^TE \succeq 0, \quad (7a-d) \]

\[ V_2(t) = \int_{-h}^{0} \int_{t+\theta}^{t} y^T(s)[R + \delta_2\bar{E}_1^T\bar{E}_1]y(s)dsd\theta. \]

The following result is obtained:

**Lemma 1** The system (4) is asymptotically stable if there exist \(n \times n\) matrices \(0 < P_1, P_2, P_3, R > 0\) and positive numbers \(\delta_1, \delta_2\) that satisfy the following LMI:

\[ \Gamma = \begin{bmatrix} \Psi & hP^T \begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} \\ P^T \begin{bmatrix} 0 \\ H \end{bmatrix} \\ hP^T \begin{bmatrix} 0 \\ H \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 \\ \bar{A}_T \end{bmatrix} \begin{bmatrix} \Psi \end{bmatrix} \begin{bmatrix} -hR \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -\delta_1I_p \\ -\delta_2hI_p \end{bmatrix} < 0 \]  

(8)

where

\[ \Psi = \Psi_0 + \begin{bmatrix} \delta_1\bar{E}_1^T\bar{E}_T \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ h(R + \delta_2\bar{E}_1^T\bar{E}_1) \end{bmatrix}, \]

\[ \Psi_0 = P^T \begin{bmatrix} 0 & I_n \\ \bar{A}_T & -I_n \end{bmatrix} + \begin{bmatrix} 0 & I_n \\ \bar{A}_T & -I_n \end{bmatrix}^T P, \]

and \(*\) denotes symmetrical entries.
Proof. Note that
\[ \bar{x}^T(t)EP\bar{x}(t) = x^T(t)P_1x(t) \]
and, hence, differentiating the first term of (6) with respect to \( t \) gives:
\[ \frac{d}{dt}\{\bar{x}^T(t)EP\bar{x}(t)\} = 2\bar{x}^T(t)P_1\dot{x}(t) = 2\bar{x}^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix}. \tag{9} \]

Replacing \[ \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix} \] by the right side of (5) we obtain:
\[ \frac{dV(t)}{dt} = \bar{x}^T(t)\Psi_0\bar{x}(t) + \eta_0 + \eta_1 + \eta_2 + hy^T(t)[R + \delta_2\bar{E}_1^T\bar{E}_1]y(t) - \int_{t-h}^{t} y^T(s)[R + \delta_2\bar{E}_1^T\bar{E}_1]y(s)ds, \tag{10} \]
where
\[ \eta_0(t) \triangleq -2\int_{t-\tau(t)}^{t} \bar{x}^T(t)P^T \begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} y(s)ds, \]
\[ \eta_1(t) \triangleq 2\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ H \end{bmatrix} \Delta(\bar{E}_0 + \bar{E}_1)x(t), \]
\[ \eta_2(t) \triangleq -2\int_{t-\tau(t)}^{t} \bar{x}^T(t)P^T \begin{bmatrix} 0 \\ H \end{bmatrix} \Delta\bar{E}_1y(s)ds. \]

Applying the standard bounding
\[ a^Tb \leq a^T Ra + b^T R^{-1} b, \quad \forall a, b \in \mathbb{R}^n, \forall R \in \mathbb{R}^{n \times n} : R > 0, \]
and using the fact that \( \tau(t) \leq h \), we have
\[ \eta_0(t) \leq \tau \bar{x}^T(t)P^T \begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} R^{-1}[0 \bar{A}_1^T]P\bar{x}(t) + \int_{t-\tau(t)}^{t} y^T(s)Ry(s)ds \]
\[ \leq h\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} R^{-1}[0 \bar{A}_1^T]P\bar{x}(t) + \int_{t-h}^{t} y^T(s)Ry(s)ds. \tag{11} \]

Similarly
\[ \eta_1 \leq \delta_1^{-1}\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ H \end{bmatrix} [0 \bar{H}^T]P\bar{x}(t) + \delta_1\bar{x}^T(t)\bar{E}_1^T\bar{E}_1x(t), \]
\[ \eta_2 \leq h\delta_2^{-1}\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ H \end{bmatrix} [0 \bar{H}^T]P\bar{x}(t) + \delta_2\int_{t-h}^{t} y^T(s)\bar{E}_1^T\bar{E}_1y(s)ds. \]

Substituting the right sides of the latter inequalities into (10), we obtain
\[ \frac{dV(t)}{dt} \leq \bar{x}^T(t)\bar{\Gamma}\bar{x}(t) \tag{12} \]
\[
\Gamma = \Psi + hP^T \begin{bmatrix}
0 \\
A_1
\end{bmatrix} R^{-1} [0 \ A_1^T] P + (\delta_1^{-1} + h\delta_2^{-1}) P^T \begin{bmatrix}
0 \\
0
\end{bmatrix} [0 \ H^T] P.
\]

Therefore, LMI (8) yields by Schur complements that \( \Gamma < 0 \) and hence \( \dot{V} < 0 \), while \( V \geq 0 \), and thus (4) is asymptotically stable [13], [4].

### 2.2 State-feedback stabilization

The results of Lemma 1 can also be used to verify the stability of the closed-loop obtained by applying (3) to the system (1) if we set in (8)

\[
\bar{A}_i = A_i + B_i K, \quad i = 0, 1, \quad \bar{E}_0 = E_0 + E_2 K
\]

and verify that the resulting LMI is feasible. The problem with (8) is that it is linear in its variables only when the state-feedback gain \( K \) is given. In order to find \( K \) we apply again Schur formula to \( \bar{\Gamma} \), the \( \Psi \) term being expanded. We thus obtain the following matrix inequality:

\[
\begin{bmatrix}
\Psi_0 & hP^T & 0 & 0 & 0 & 0 \\
0 & hI_n & 0 & 0 & 0 & 0 \\
0 & 0 & -hR^{-1} & 0 & 0 & 0 \\
0 & 0 & 0 & -\delta_1^{-1}I_q & 0 & 0 \\
0 & 0 & 0 & 0 & -\delta_2^{-1}hI_q & 0 \\
0 & 0 & 0 & 0 & 0 & -\delta_2^{-1}hI_p
\end{bmatrix} < 0
\]

Consider the inverse of \( P \). It is obvious, from the requirement \( P_1 > 0 \) and the fact that in (8) \(-(P_3 + P_3^T)\) must be negative definite, that \( P \) is nonsingular. Defining

\[
P^{-1} = Q = \begin{bmatrix}
Q_1 & 0 \\
Q_2 & Q_3
\end{bmatrix} \quad \text{and} \quad M = \text{diag}(Q, I_{2(n+p+q)})
\]

we multiply (14) by \( M^T \) and \( M \), on the left and on the right, respectively. Choosing

\[
R^{-1} = Q_1 \varepsilon,
\]
where $\varepsilon$ is a positive number, and introducing $\bar{\delta}_1 = \delta_1^{-1}$ and $\bar{\delta}_2 = \delta_2^{-1}$, we obtain the LMI

$$
\begin{bmatrix}
\Phi & h \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} -hQ_1\varepsilon & 0 \\
\bar{A}_1 Q_1 \varepsilon & hI \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} Q^T \end{bmatrix} \begin{bmatrix} \bar{E}^T \end{bmatrix} \\
* & -hQ_1\varepsilon & 0 \\
* & * & -hQ_1\varepsilon & 0 \\
* & * & * & -\bar{\delta}_1 I_q \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix} < 0
\end{bmatrix} (16)
$$

where

$$
\Phi = \begin{bmatrix} 0 & I_n \\
\bar{A}_T & -I_n \end{bmatrix} Q + Q^T \begin{bmatrix} 0 & I_n \\
\bar{A}_T & -I_n \end{bmatrix}^T.
$$

Substituting (13) into (16) and denoting $Y = KQ_1$, $B_T = B_0 + B_1$, we obtain

**Theorem 1** The control law of (3) asymptotically stabilizes (1) if, for some positive number $\varepsilon$, there exist scalars $\bar{\delta}_1 > 0$, $\bar{\delta}_2 > 0$ and matrices $0 < Q_1, Q_2, Q_3, \in \mathbb{R}^{n \times n}$ $Y \in \mathbb{R}^{m \times n}$ that satisfy the following LMI:

$$
\begin{bmatrix}
Q_2 + Q_2^T & Q_1 A_1^T + Y^T B_1^T - Q_2^T + Q_3 & 0 & hQ_2^T \\
* & -Q_3 - Q_3^T & h\varepsilon (A_1 Q_1 + B_1 Y) & hQ_3^T \\
* & * & -h\varepsilon Q_1 & 0 \\
* & * & * & -hQ_1\varepsilon \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
\end{bmatrix} (17)
$$
\[
\begin{pmatrix}
Q_1 E_1^T + Y^T E_2^T & h Q_2^T E_1^T & 0 & 0 \\
0 & h Q_3^T E_1^T & \bar{\delta}_1 H & h \bar{\delta}_2 H \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-\bar{\delta}_1 I_q & 0 & 0 & 0 \\
* & -h \bar{\delta}_2 I_q & 0 & 0 \\
* & * & -\bar{\delta}_1 I_p & 0 \\
* & * & * & -\bar{\delta}_2 I_p \\
\end{pmatrix} < 0
\]

(18)

The state-feedback gain is then given by

\[
K = Y Q_1^{-1}.
\]

(19)

3 Sliding mode controller

In this section, we focus on time-delay systems that can be represented, possibly, after a change of state coordinates and input, in the following regular form ([9],[18]):

\[
\begin{align*}
\dot{z}_1(t) &= (A_{11} + H \Delta(t) E_0) z_1(t) + (A_{d11} + H \Delta(t) E_1) z_1(t - \tau(t)) \\
\dot{z}_2(t) &= (A_{12} + H \Delta(t) E_2) z_2(t) + A_{d12} z_2(t - \tau(t)) \\
z(t) &= \phi(t) \text{ for } t \in [-h, 0]
\end{align*}
\]

(20)

where \( z(t) = (z_1, z_2)^T, z_1 \in \mathcal{R}^{n-m}, z_2 \in \mathcal{R}^m, A_{ij}, A_{dij}, i = 1, 2, j = 1, 2, E_k, k = 0, 1, 2, H \) are constant matrices of appropriate dimensions, \( D \) is a regular \( m \times m \) matrix, the matrix \( \Delta(t) \) is a time-varying matrix of uncertain parameters, \( u \in \mathcal{R}^m \) is the input vector, \( \tau \) is time-varying delay satisfying \( 0 \leq \tau(t) \leq h, \forall t \geq 0, \) \( z_i(\theta) \) is the function associated with \( z \) and defined on \([-h, 0]\) by \( z_i(\theta) = z(t + \theta), \phi \) is the initial piecewise continuous function defined on \([-h, 0]\).

We will assume that:

A1) \( (A_{11} + A_{d11}, A_{12} + A_{d12}) \) is controllable.

A2) \( f \) is Lipschitz continuous and satisfies the inequality

\[
\|f(t, z_t)\| < F_M(t, z_t), \quad \forall t \geq 0,
\]

where \( F_M(t, z_t) \) is a continuous functional assumed to be known \textit{a priori},

A3) \( \Delta(t) \) is a time-varying matrix of uncertain parameters satisfying \( \Delta^T(t) \Delta(t) \leq I \) \( \forall t \).

Consider the following switching function:

\[
s(z) = z_2 - K z_1
\]

(21)
with $K \in \mathcal{R}^{m \times (n-m)}$. Let $\Omega$, $\Theta$ be the linear functions defined by

$$
\Omega(z(t)) = \sum_{i=1}^{2} (A_{2i} - KA_{1i})z_i(t),
$$
$$
\Theta(z(t)) = E_0z_1(t) + E_2z_2(t)
$$

and let $D_M$ be the following functional:

$$
D_M(z_t) = (\|A_{d21} - KA_{d11}\| + \|KH\| \|E_1\|) \sup_{-h \leq \theta \leq 0} \|z_1(t + \theta)\|
$$
$$
+ \|A_{d22} - KA_{d12}\| \sup_{-h \leq \theta \leq 0} \|z_2(t + \theta)\|.
$$

Following [9] and using the results of previous section, we are able to design a sliding mode controller that will stabilize system (20) under less conservative assumptions on the delay law.

**Theorem 2** Assume $A1$-$A3$. If, for some positive number $\varepsilon$, there exist positive numbers $\bar{\delta}_1, \bar{\delta}_2$ and matrices $0 < Q_1, Q_2, Q_3 \in \mathcal{R}^{(n-m) \times (n-m)}, Y \in \mathcal{R}^{m \times (n-m)}$ that satisfy the following LMI:

$$
\begin{bmatrix}
Q_2 + Q_2^T & X_{12} & 0 & hQ_2^T \\
* & -Q_3 - Q_3^T & h\varepsilon(A_{d11}Q_1 + A_{d12}Y) & hQ_3^T \\
* & * & -h\varepsilon Q_1 & 0 \\
* & * & * & -h\varepsilon Q_1 \\
* & * & * & * \\
* & * & * & * \\
* & * & * & * \\
Q_1E_1^T + Y^TE_2^T & hQ_3^TE_1^T & 0 & 0 \\
0 & hQ_3^TE_1^T & \bar{\delta}_1 H & h\bar{\delta}_2 H \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-\bar{\delta}_1 I & 0 & 0 & 0 \\
* & -h\bar{\delta}_2 I & 0 & 0 \\
* & * & -\bar{\delta}_1 I & 0 \\
* & * & * & -\bar{\delta}_2 hI
\end{bmatrix} < 0.
$$

where

$$X_{12} = Q_1(A_{11}^T + A_{d11}^T) + Y^T(A_{12}^T + A_{d12}^T) - Q_2^T + Q_3,$$

then the sliding mode control law

$$
u(t) = -D^{-1} \left[ \Omega(z(t)) + (F_M(t, z_t) + D_M(z_t) + \|KH\| \|\Theta(z(t))\| + M) \frac{s(z(t))}{\|s(z(t))\|} \right],
$$

where $K = YQ_1^{-1}$, $M > 0$ and $s, \Omega, \Theta, D_M$ are defined in (21)-(23), asymptotically stabilizes system (20) for any delay function $\tau(t) \leq h$. 

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**Proof:** The proof is divided into two parts. The first one is dedicated to the proof of the existence of an ideal sliding motion on the surface \( s(z) = 0 \), the second part to the proof of the stability of the reduced system.

**Attractivity of the manifold:**
Consider the Lyapunov-Krasovskii functional
\[
V(t) = s^T(z(t))s(z(t)) = \|s(z(t))\|^2. \tag{26}
\]
Differentiating (26) on the trajectories of the closed-loop system gives
\[
\dot{V}(t) = 2s^T(t)(\Omega(z(t)) + \sum_{i=1}^{2}[A_{d2i} - KA_{d1i}]z_i(t - \tau) + Du(t) + f(t, z_i) - KH\Delta(t)[\Theta(z(t)) + E_1z_1(t - \tau(t))]),
\]
Using the expression of the control law (25), we get
\[
\dot{V}(t) = 2s^T(t)\left(\sum_{i=1}^{2}(A_{d2i} - KA_{d1i})z_i(t - \tau) + f(t, z_i) - KH\Delta(t)[\Theta(z(t)) + E_1z_1(t - \tau(t))] - \left[F_M(t, z_i) + D_M(z_i) + \|KH\| \|\Theta(z(t))\| + M\right]8\right)
\]
then we derive that:
\[
\dot{V} \leq -2M \|s(z(t))\| = -2M\dot{V}(t)^\frac{1}{2}.
\]
This last inequality is known to prove the finite-time convergence of the system (20) into the surface \( s = 0 \) ([18]).

**Stability of the reduced system:**
On the sliding manifold \( s(z) = 0 \), the system is driven by the following reduced system:
\[
\frac{dz_1(t)}{dt} = (A_{11} + A_{12}K + H\Delta(t)(E_0 + E_2K))z_1(t) + (A_{d11} + A_{d12}K + H\Delta(t)E_1)z_1(t - \tau(t)) \tag{27}
\]
According to Theorem 1, this system is asymptotically stable for any delay law \( \tau(t) \leq h \) if, for some positive number \( \varepsilon \), there exist positive numbers \( \delta_1, \delta_2 \) and matrices \( 0 < Q_1, Q_2, Q_3, Y \in \mathbb{R}^{m\times(n-m)} \) that satisfy the LMI (24).

**Remark 1** Note that the explicit knowledge of the time-dependance of the delay is not required in the expression of the control law \( u(t) \), all is needed is the knowledge of an upper bound \( h \).

## 4 Example

We demonstrate the applicability of the above theory by solving the example from [9] for a system without uncertainty. Consider system
\[
\dot{x}(t) = Ax(t) + A_d(x(t - \tau) + B[u(t) + f(x, t)], \tag{28}
\]
Table 1: Comparison of results for example (27)-(28)

<table>
<thead>
<tr>
<th></th>
<th>delay upper bound</th>
<th>type of delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theorem 2</td>
<td>3.999</td>
<td>time-varying</td>
</tr>
<tr>
<td>Gouaisbaut et al [9]</td>
<td>1.65</td>
<td>constant</td>
</tr>
<tr>
<td>Ivanescu et al.[10]</td>
<td>1.46</td>
<td>constant</td>
</tr>
<tr>
<td>Fu et al.[8]</td>
<td>0.984</td>
<td>constant</td>
</tr>
<tr>
<td>Li and de Souza[14]</td>
<td>0.51</td>
<td>constant</td>
</tr>
</tbody>
</table>

with a time-varying delay, where

\[
A = \begin{bmatrix}
2 & 0 \\
1.75 & 0.25
\end{bmatrix}, \quad A_d = \begin{bmatrix}
-1 & 0 \\
-0.1 & -0.25
\end{bmatrix}, \quad B = \begin{bmatrix}
1 \\
1
\end{bmatrix}.
\] (29)

By an appropriate change of variables, this system is equivalent to:

\[
\dot{z}(t) = \tilde{A}z(t) + \tilde{A}_d z(t - \tau) + \tilde{B}[u(t) + f(x, t)],
\]

where

\[
\tilde{A} = \begin{bmatrix}
0.25 & 0 \\
1.75 & 2
\end{bmatrix}, \quad \tilde{A}_d = \begin{bmatrix}
-0.9 & -0.65 \\
-0.1 & -0.35
\end{bmatrix}, \quad \tilde{B} = \begin{bmatrix}
0 \\
1
\end{bmatrix}.
\] (30)

As the pair \((\tilde{A}_{11}, \tilde{A}_{12})\) is not controllable, the system cannot be stabilized independently of the delay.

For this system, previous published works give the following results:

— In the case of a constant delay and \(f = 0\), the system may be stabilized using a linear memoryless controller \(u(t) = Kx(t)\) for the following maximum values of \(h\): \(h = 0.51\) by [14], \(h = 0.984\) by [8] and \(h = 1.46\) by [10]. By sliding mode control for the case of constant delay and \(f \neq 0\) the maximum value of \(h = 1.65\).

— Applying Theorem 2 in the case of a time-varying delay and \(f \neq 0\), the corresponding value of \(h = 3.999\) is achieved.

This is summarized in table 1.

5 Conclusions

The problem of finding a sliding mode controller that asymptotically stabilizes a system with time-varying delay and norm-bounded uncertainty has been solved. A delay-dependent solution has been derived using a special Lyapunov-Krasovskii functional. The result is based on a sufficient condition
and it thus entails an overdesign. This overdesign is considerably reduced due to the fact that the
method is based on the descriptor representation. As a byproduct for the first time on the basis
of the descriptor model transformation the solution to the stabilization problem by the feedback,
which depends on both, non-delayed and delayed state is solved. Finally, a numerical example
shows the effectiveness of the combined method: sliding mode and descriptor representation.

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