Networked Control Systems:

to buff, or not to buff?

Jean-Pierre Richard

http://researchers.lille.inria.fr/~jrichard
jean-pierre.richard@ec-lille.fr

GREYC, Caen 27 novembre 2013

* Systèmes Contrôlés en Réseau : s’en tamponner, ou pas ?
**NCS: to **buff**, or not to buff?**

Networks such as Internet or Wireless 802.11 present great advantages for flexible and low-cost networking. However, they are not as reliable as CANs, and integrating them in control applications, while preserving some performance, constitutes an interesting challenge.

Using delay models allows for catching many of the effects introduced by the presence of unreliable networks in the control loops. Several theoretical techniques allow for analyzing the resulting systems. Some of them need the delay to be constant, which can be obtained by using waiting strategies involving buffers. Some other tolerate fast varying delays.

Without entering into many technical details, we'll try to draw a panel of some techniques, in particular the ones we developed at LAGIS.

*It will be some story about network effects, sampling and lost packets, time /event -driven solutions, remote observers and bilateral teleoperation...*

**synchrone / asynchrone**
Verbe

buff /ˈbʌf/ 
1. Polir, poncer, meuler, buffler.
2. (Jeux vidéo) Rendre un personnage plus fort.

Antonymes

Nom commun

buff /ˈbʌf/ 
1. Cuir brut obtenu de la peau du buffle ou d'un animal similaire.

Nom commun

buffer /ˈbʌfər/ 
1. (Transport) Tampon, pare-chocs.
2. (Chemin de fer) (Royaume-Uni) Heurtoir (train buffer stop).
3. (Informatique) Mémoire tampon, tampon (data buffer).
   - Buffer overrun is a common cause of bugs.
4. (Informatique) Tampon d'affichage (display buffer).
5. (Chimie) Solution stabilisant le pH d'un liquide.
Overview

- Motivation and examples
- Modelling
- Sampling and delay
- A crude example
- Control: *to buff, or not to buff?* A selection of results
- Conclusions
Tele surgery: the Lindbergh operation, 07/09/2001

Constant RTT
< 200 msec
Distance:
17 000 km
Com. cost ≈ 160k$/month

« The only restriction to the development of long-distance tele-surgery as to do, still today, with its cost. For tele-surgery, you must use a transcontinental ATM line, that you have to book during 6 months, at the price of about 1 million dollars. »

Prof. J. Marescaux, Le Monde, January 6, 2010
other examples…

Movies: see http://chercheurs.lille.inria.fr/~jrichard/manips.htm
and an example from everyday life…

TO BE OR NOT TO BE, THAT IS THE QUESTION: WHETHER 'TIS NOBLER IN THE MIND TO SUFFER THE SLINGS AND ARROWS OF OUTRAGEOUS FORTUNE, OR TO TAKE ARMS AGAINST A SEA OF TROUBLES, AND BY OPPOSING, END THEM? TO DIE: TO SLEEP; NO MORE; AND BY A SLEEP TO SAY WE END

Natural loop of audio-phonatory control
and an example from everyday life...

Networked loop of audio-phonatory ctrl
Some surveys…

- Control methodologies in Networked Control Systems

- Networked Control System: a brief survey

- Control over Wireless Networks

- A survey of recent results in Networked Control Systems

- Trends in Networked Control Systems
  S. Zampieri, 17th IFAC World Congress, Seoul, 2008

- A switched system approach to exponential stabilization through com. network
not forgetting…

Systèmes commandés en réseau

sous la direction de
Jean-Pierre Richard
Thierry Divoux

2007
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Single-loop NCS

What about multi-loop?

Tipsuwan 2003
Multiple-loop NCS according to [Yang 2006]:

Fig. 1 Typical NCS setup and information flows

Fig. 2 is a block diagram, representing a general framework for the study of networks and control. An NCS, which is the main topic of this paper, can be considered as having two subsystems interacting with each other through networked communication channels. Here, it is

Fig. 2 General framework for networks and control

Yang 2006
Multiple-loop NCS according to [Hespanha 2007]:

It is also often common to consider a single feedback loop as in Fig. 2. Although considerably simpler than the system shown in Fig. 1, this architecture still captures many important characteristics of NCSs, such as bandwidth limitations, variable communication delays, and packet dropouts.
Single-loop NCS.

... the talk will be limited to this case, too.

A good starting point for understanding the issues linked to the presence of networks in the loops.
Types of networks:

- **dedicated** (ControlNet, DeviceNet): frequent transmission of small packets $\rightarrow$ guaranteed time but €
- Ethernet, wifi: rare transmission of bigger packets $\rightarrow$ no guarantee for delay but €
## Effects of the network on the closed-loop control

<table>
<thead>
<tr>
<th>Issue</th>
<th>Translation in control terms</th>
<th>Concerned</th>
<th>Not concerned</th>
</tr>
</thead>
<tbody>
<tr>
<td>limited bandwidth</td>
<td>quantification, limited quantity of info per second (Shannon, <em>maximum bit rate</em>)</td>
<td>limited energy resource systems (UAVs, WSN, µ-sensors or µ-actuators, aerosp…)</td>
<td>packet-transmiss. type Inter/Ethernet, Bluetooth… 1 bit or 300 → <em>idem</em></td>
</tr>
<tr>
<td>sampling, coding, scheduling, transmission, asynchronism</td>
<td>variable delays, estimated if there is a model, or time-stamps</td>
<td>packet transmission systems</td>
<td>dedicated and unshared netw. (ControlNet, DeviceNet)</td>
</tr>
<tr>
<td>packet losses</td>
<td>asynchronous sampling, variable delay</td>
<td>wireless, UDP-like protocols</td>
<td>TCP-like protocols (but generally useless: time wasting for outdated info)</td>
</tr>
<tr>
<td>out-of-sync clocks</td>
<td>delays (at least)</td>
<td>internet</td>
<td>control-dedicated netw. (CAN bus…</td>
</tr>
</tbody>
</table>
In such a framework, we have the equation:

\[ \text{communication} + \text{packet loss} + \text{sampling} = 1 \text{ delay} \]
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Sampled and hold signal
(depicted for a constant period)

Delayed signal with variable $h(t)$

\[ u(t) = u_d(t_k) = u_d(t - [t - t_k]) = u(t - h(t)) \]
A periodic sampling

$u(t) = u_d(t_k) = g(x(t_k)), \quad t_k \neq kT$

Integration, $z$-transform, everything’s ok, etc.
Another statement of the same problem...

- influence of the maximum sampling period $h$
- application of Fridman’s criterion $\dot{h}(t) \leq 1$

See also refined results in:

“A refined input delay approach to sampled-data control”. E. FRIDMAN, Automatica 2010
“A novel stability analysis of linear systems under asynchronous sampling”. A. SEURET, Automatica 2011
“SOS for sampled-data systems” A. SEURET, M. PEETS, IFAC’11, Milano, Italy, 2011
Note: approaching aperiodic sampling via *switched systems*

PhD L. Hetel 2007 + IEEE TAC 2006 (Daafouz, Iung)

intra-sampling: Taylor trunc. of the exponential terms (cont. delay) \(\rightarrow\) *polytopes, LMI*

nb of packet losses \(\rightarrow\) discrete delay

augmented model \(\rightarrow\) *switched system* (event-based model)

\[
x(k + 1) = A(k)x(k) + B(k)u(k)
\]

\[
A(k) = e^{M(t_{k+1} - t_k)}, \quad B(k) = \int_0^{t_{k+1} - t_k} e^{M(t_{k+1} - t_k - s)} dsN.
\]

\[
\eta_{i+1} = A(\rho_i)\eta_i + B(\rho_i)u_i.
\]

\[
A(\rho_i) = e^{M\rho_i}, \quad B(\rho_i) = \int_0^{\rho_i} e^{Ms} dsN.
\]

\[
u_c(t_i) = Kx_c(t_i) = Kx_c(t_{i-\theta_i}),
\]

\[
\theta_i \in T = \{\theta \in \mathbb{Z}^+: \theta \leq \theta_i \leq \bar{\theta}\}
\]

\[
z_i = \begin{bmatrix} \eta_i^T & \eta_{i-1}^T & \cdots & \eta_{i-\theta_i}^T \end{bmatrix}^T \rightarrow \bar{A}(\rho_i) = \begin{bmatrix} A(\rho_i) & 0 & \cdots & 0 \\ I & 0 & \cdots & 0 \\ 0 & I & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & I & 0 \end{bmatrix}, \quad \bar{B}(\rho_i) = \begin{bmatrix} B(\rho_i) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}
\]

\[
u_i = \bar{K}(\theta_i)z_i,
\]

\[
\bar{K}(\theta_i) = [0 \quad \cdots \quad 0 \quad K \quad 0 \quad \cdots \quad 0]
\]

\[
z_{i+1} = (\bar{A}(\rho_i) + \bar{B}(\rho_i)\bar{K}(\theta_i))z_i
\]
Note: approaching aperiodic sampling via \textit{switched systems}

**Unification of both approaches  input delay / switched:**

"Discrete and intersample analysis of systems with aperiodic sampling"

L. HETEL, A. KRUSZEWSKI, W. PERRUQUETTI, J.P. RICHARD


\[
\dot{x}(t) = A_c x(t) + B_c K x(t_k), \quad \forall t \in [t_k, t_{k+1}]
\]

\[
x(t) = \Lambda(t - t_k) x(t_k)
\]

\[
\Lambda(\theta) := I + \int_0^\theta e^{s A_c} ds (A_c + B_c K)
\]

\[
\mathcal{H}(x) = \{y : y = \Lambda(\theta)x, \theta \in T\}
\]

\[
x(t_{k+1}) \in \mathcal{H}(x(t_k))
\]

\[
\exists \text{ quasi-quadratic Lyapunov function}
\]

\[
V(x) = x^T \mathcal{L}[x] x, \quad \mathcal{L}[:,] : \mathbb{R}^n \to \mathbb{R}^{n \times n},
\]

\[
\mathcal{L}[x] = \mathcal{L}[x] = \mathcal{L}[ax], \quad \forall x \neq 0, a \neq 0
\]

\[
x = 0 \text{ asympt. stable}
\]

\[
\text{constructive}
\]
Note: State-Dependent Sampling

**Next sampling instant depends on state** \( x_k \)

"A state-dependent sampling for linear state feedback"

C. FITER, L. HETEL, W. PERRUQUETTI, J.P. RICHARD

*Automatica* 48 (8), p.1860-1867, 2012

\[
\dot{x}(t) = A_c x(t) + B_c K x(t_k), \quad \forall t \in [t_k, t_{k+1}]
\]

\[
\dot{x}(t) = \begin{pmatrix} 1 & 15 \\ -15 & 1 \end{pmatrix} x(t) - \begin{pmatrix} 1 \\ 1 \end{pmatrix} K x(t_k),
\]

\[ K = \begin{pmatrix} -5.33 \\ 9.33 \end{pmatrix}. \]

\[ t_{k+1} = t_k + \tau(x(t_k)) \]

Max next sampling computed off-line (\( \beta \)-stab.) \( \rightarrow \) less often than with periodic sampling
... let’s sum up until now:

transmission time + access time + packet loss + sampling…

= 2 variable delays

known / unknown?

Hyp: Clock Synchro NTP, GPS
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A crude example

Theory (when neglecting the network effects)

\[
\begin{align*}
\dot{x}(t) & = \begin{bmatrix} 0 & 1 \\ 0 & -10 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.024 \end{bmatrix} u(t), \\
y(t) & = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t).
\end{align*}
\]

controller poles = -1 and -15
observer poles = -2 and -20

\[
K = \begin{bmatrix} -750 & -300 \end{bmatrix} \quad L = \begin{bmatrix} 12 \\ -80 \end{bmatrix}
\]
A crude example

Experimental results (with network)

LTI model with order 2
state feedback gain \( K \)
and observer gain \( L \)
computed by pole assignement
without paying attention to the net
(here, Internet over 40km)

\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & -10 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.024 \end{bmatrix} u(t), \\
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\]
A crude example

Interpretation on a simplified model

**target angle**
\[ x^c = 0 \]

**gap**
\[ \varepsilon = 0 - x \]

**voltage** \( u \)

**drive**

**measured angle** \( x \)

**speed**
\[ \dot{x}(t) = ku(t), \quad \frac{X(s)}{U(s)} = \frac{k}{s} \]

**feedback**:
\[ \dot{x}(t) = ku(t) = -kx(t) \]

\[ \dot{x}(t) + kx(t) = 0 \]

\( ODE \)
A crude example

remote feedback:
\[ u(t) = k \varepsilon(t - h/2) = -kx(t - h) \]

measured angle \( x \)

transmitted angle \( x(t-h/2) \)

transmitted control \( \varepsilon(t-h/2) \)

\[ \dot{x}(t) + kx(t - h) = 0 \]
A crude example

Exercise... for my students, don’t worry ;-)

\[ \dot{x}(t) + x(t - h) = 0 \]  
(cas $h = 1, k = 1$)

\[ \dot{x}(t) = -x(t - 1) \]

C.I. $t = 0 : x(t = 0) = 1$ ??

$t \in [-1, 0] : x(t) = 1$ (C.I.)

$t \in [0, 1] : x(t) = 1 - t,$

$t \in [1, 2] : x(t) = \frac{1}{2} - t + \frac{t^2}{2}, 0$

etc.
A crude example

\[ \dot{x}(t) + x(t - h) = 0 \]  

w.r.t. \ h \ ?

\[ \dot{x}(t) + x(t) = 0 \]

\[ \dot{x}(t) + x(t - 1) = 0 \]

\[ \dot{x}(t) + x(t - \frac{\pi}{2}) = 0 \]  

\[ \dot{x}(t) + x(t - 1.6) = 0 \]
\[ \frac{dx}{dt} = -ax(t) - bx(t - h(t)) \] (1)

\[ h(t) = t - kT \quad \text{for} \quad kT < t \leq (k + 1)T \]

\[ \begin{array}{c}
\text{variable: asymptotically stable iff } \in \text{ yellow zone:} \\
|1 + \frac{b}{a}e^{-aT} - \frac{b}{a}| < 1 \quad \text{if } a \neq 0 \\
|1 - bT| < 1 \quad \text{if } a = 0
\end{array} \]

\[ (T = 1) \]

\[ \begin{array}{c}
\text{constant: } \forall h \in [0,1] \quad \text{iff } \in \text{ grey zone}
\end{array} \]
A crude example

A known delay may also have a stabilizing effect

Here, derivative effect: \[ y(t - h) \approx y(t) - h \dot{y}(t) \]
Few words about Lyapunov method for TDS

ODE:
\[ \dot{x}(t) = -ax(t) \rightarrow V(x(t)) = x^2(t) > 0 \]
\[ \dot{V}(x(t)) = -2ax^2(t) < 0 \ldots \text{ etc.} \]

FDE:
\[ \dot{x}(t) = -ax(t) - bx(t-h) \]
\[ V(x(t)) = x^2(t) \quad \text{("usual" quadratic)} \]
\[ \dot{V}(x(t)) = -2 \left[ ax^2(t) + bx(t)x(t-h) \right] \leq \ldots \text{ ?} \]

→ need of specific adaptations:
1) functions of Lyapunov-Razumikhin (not here)
2) functionals of Lyapunov-Krasovskii
Illustration of the LKF approach

(Lyapunov-Krasovskii functionals)

\[
\dot{x}(t) = -ax(t) - bx(t - h)
\]

\[
V(x_t) = x^2(t) + |b| \int_{-h}^{0} x^2(t + s) \, ds
\]

\[
\dot{V}(x_t) = -2x(t) [ax(t) + bx(t - h)]
+ |b| [x^2(t) - x^2(t - h)]
\leq -2(a - |b|) x^2(t) \quad \cdots \quad \dot{V}(x_t) < 0 \text{ if } |b| < a
\]

Remarks on the conservatism:
1) here, a « pessimistic » condition since it is i.o.d. - independent of the delay \( h \) ...
2) linked with the majoration of the crossed terms, here :

\[-[x(t) + x(t - h)]^2 \leq 0 \]

\[\Rightarrow -2bx(t)x(t - h) \leq bx^2(t) + bx(t - h)^2\]
Lyapunov for TDS

More general...

The system model

$$\dot{x}(t) = Ax(t) + A_1 x(t - \tau(t)), \quad (2)$$

with the initial condition:

$$x(t_0 + \theta) = \phi(\theta), \quad \dot{x}(t_0 + \theta) = \dot{\phi}(\theta), \quad \theta \in [-h_2, 0], \quad (3)$$

where $\tau(t) \in [h_1, h_2], \quad h_1 \geq 0$.

Lyapunov functionals

$$V(t, x_t, \dot{x}_t) = x^T(t)Px(t) + \int_{t-h_1}^{t} x^T(s)Sx(s)ds$$
$$+ h_1 \int_{-h_1}^{0} \int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta$$
$$+ \int_{t-h_2}^{t} \dot{x}^T(s)S_a\dot{x}(s)ds + (h_2 - h_1) \int_{-h_2}^{t} \int_{t+\theta}^{t} \dot{x}^T(s)R_a\dot{x}(s)dsd\theta$$

where $P > 0$ and $R, R_a, S, S_a \geq 0$. 
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to buff, or not buff?

1\textsuperscript{rst} solution = act « as if » constant
✓ [Niemeyer & Slotine 98][Huang & Lewis 03][Azorin et al. 03][Fattouh & Sename 03] etc.

2\textsuperscript{nd} solution = force the delay to constant
- therefore, maximize it: \(0 \leq h_i(t) \leq h_{\text{max}} \Rightarrow h_i(t) = h_{\text{max}}\)
- thanks to a \textit{buffer} → \textit{time-driven}
- then, apply classical techniques:
  ✓ prediction (Smith) [Lelevé & Fraisse 01]
  ✓ error eqn. obeying a retarded model [Estrada, Marquez, Moog 07]
  ✓ etc.

3\textsuperscript{rd} solution (intermed.) only one buffer
[Seuret 06] [Jiang et.al 08]

4\textsuperscript{th} solution = keep the variable delay…
✓ [Witrant et al. 07] [Seuret&Richard 08] → \textit{event-driven}
  [Jiang et al. 09] [Kruszewski et al. 11, 12]

\textit{Network delay: variable, asymmetric}
4.1. Control with predictive model of the network

PhD E. Witrant 2005 + IEEE TAC 2007 (Witrant, Canudas, Georges, Alamir)

with a model of the network and without packet loss (predictable delay)

- state predictor with variable delays (implicit equation)
- controller of FSA type
- variable prediction horizon
- proven robustness (via small gain)
- Application to the T inverted pendulum

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t - \tau(t)), \\
y(t) &= Cx(t) \\
\dot{z}(t) &= f(z(t), u_d(t)), \quad z(0) = z_0 \\
\tau(t) &= h(z(t), u_d(t))
\end{align*}
\]

where \(\hat{\delta}(t) = \hat{\tau}(t + \hat{\delta}(t))\) is the prediction horizon.

Time-delay on the actuator (a) and measurement (b) signals.
Chapitre 5. Application et Expérimentation

Retard induit par le réseau

Fig. 5.6 – Banc d’essai expérimental.
4.2. Control with buffers (two-way)

Aim: synchronization of a Slave pendulum (Nantes) on a Master one (Ensenada).

\( y_{\text{ref}} \) supposed t.b. known from both sides.

Strategy:

1) buffers at 300ms = \( \tau \)

2) control s.t. M/S error obeys:

\[
e^{(3)}(t) + a_2 \dot{e}(t-\tau) + b_2 \ddot{e}(t-\tau) + c_2 e(t-\tau) = 0.
\]

\( \rightarrow \) causal control

simulated pendulum, real network

real pendulum, real network
4.3. Commande avec tampon aller seul

PhD A. Seuret 2006  (+Dambrine, Richard)

Master

-set value

Controller

Observer

GPS/NTP

Zero-Holder

Gain $K$

Gain $L$

Network

UDP

Slave

Implementation card

Zero-Holder

Sensors

GPS/NTP

buffer

$h_1$

$h_2$

$	au_1$

$	au_2$

\[
\begin{align*}
\dot{x}(t) &= A\hat{x}(t) + Bu(t - \delta_1(t)) - L(y(t - \delta_2(t)) - \hat{y}(t - \delta_2(t))), \\
\dot{y}(t) &= C\hat{x}(t),
\end{align*}
\]

- defines the target
- receives Slave’s output
- observes Slave’s state
- computes & sends control

-known thx to buffer

Limited computation power

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t - \delta_1(t)), \\
y(t) &= Cx(t).
\end{align*}
\]

-known thx to time-stamps

-receives & applies control

-sends measured output
Passage à l’expérimental + adaptation à la QdS disponible

Master → Internet → Slave PC → Bluetooth → Slave Miabot

Internet:
- Nighttime: 120 ms
- Daytime: 60 ms

Bluetooth:
- Time delay: 60 ms
- The maximum delay: 51.77 ms
- The minimum delay: 35.75 ms
- The average delay: 37.50 ms
One week of RTT…

RTT (40km)
Mean = 82 ms
Maxi = 857 ms
Mini = 1 ms

Lille-Lens (40 km) – Source: WJ. Jiang 2008
RTT (1640km)
Maxi = 415 ms
Mini = 70 ms

Other RTT (approx. values):
- unshared CAN 2m: 200µsec
- internet 40km: 300ms
- orbital stations: 0.4-7s
- bluetooth 2m: 40ms
- internet 1640km: 300ms
- underwater 1.7km: >2sec
One week of RTT...

The achievable exponential rate depends on $h_m$, which motivates some adaptation w.r.t. QoS...

$h_m = 0.05s, \alpha = 8.74$

$h_m = 0.5s, \alpha = 0.96$

Link max delay $\leftrightarrow$ provable performance
Model of the switching system

Two switching modes are considered: the big time-delay and the small time-delay.

1) \[
\dot{x}(t) = Ax(t) + \chi_{[h_1, h_2]}(\delta_{\text{con}}(t))BK_1 x(t - \delta_{\text{con}}(t)) \\
+ (1 - \chi_{[h_1, h_2]}(\delta_{\text{con}}(t)))BK_2 x(t - \delta_{\text{con}}(t)),
\]

2) \[
\dot{e}(t) = Ae(t) - \chi_{[h_1, h_2]}(\delta_{\text{obs}}(t))L_1 Ce(t - \delta_{\text{obs}}(t)) \\
- (1 - \chi_{[h_1, h_2]}(\delta_{\text{obs}}(t)))L_2 Ce(t - \delta_{\text{obs}}(t)).
\]

\(\chi : \mathbb{R} \rightarrow \{0, 1\}\) is defined by:

\[
\chi_{[h_1, h_2]}(s) = \begin{cases} 
1, & \text{if } s \in [h_1, h_2] \\
0, & \text{otherwise} 
\end{cases}
\]

General switched system

\[
\dot{x}(t) = Ax(t) + \chi_{[h_1, h_2]}(\tau)A_1 x(t - \tau(t)) + (1 - \chi_{[h_1, h_2]}(\tau))A_2 x(t - \tau(t)),
\]

The LKF:

\[
V(t, x_t, \dot{x}_t) = x^T(t)Px(t) + \sum_{i=0}^{2} \int_{t-h_i+1}^{t} x^T(s)S_i x(s) ds \\
+ \sum_{i=0}^{2} (h_{i+1} - h_i) \int_{t+\theta}^{t} \int_{t+h_i}^{-h_i} x^T(s)R_i \dot{x}(s) ds d\theta,
\]

where \(h_0 = 0\), \(P > 0\) and \(R_i, S_i \geq 0\).
buffer and gains $K$, $L$ adapted to the QoS (delay ranges)

Note the predictor effect (compensates the network delay)
4.3. Buffer-free control \textit{(event-driven)}

A. Seuret - JPR 2008 (theory) + W. Jiang, A. Kruszewski \textit{et al.} (experim.+switches) 

cf. IEEE T. CST 2011

\begin{equation}
\begin{cases}
\dot{x}(t) = A\dot{x}(t) + Bu(t - \delta_1(t)) \\
- L(y(t - \delta_2(t)) - y(t - \delta_2(t))), \\
\hat{y}(t) = C\dot{x}(t),
\end{cases}
\end{equation}
Results from the event-driven mode

Event-driven vs. time-driven mode.

(without buffer)

(with buffer)
Natural continuation: teleoperation \((LKF + H_\infty)\)

1) Basic scheme: position tracking - \(\theta\) (no force target)

\[
w(t) = \begin{pmatrix} F_e(t) \\ F_h(t) \end{pmatrix}
\]

Network

\[
\begin{align*}
\tau_1 &= w(t) \\
\tau_2 &= \dot{x}(t) = \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_m(t) \\ \theta_s(t) - \theta_m(t) \end{pmatrix}
\end{align*}
\]

PhD Bo Zhang (2012) (+ Kruszewski, Richard)
2) Improved scheme: force and position tracking (haptic rendering) (ETFA 2011)

- Proxy synchronized on Master
- Slave synchronized on Proxy

• Reduction of the delay effect
• Accurate haptic rendering
2) Improved scheme: force and position tracking (haptic rendering) (ETFA 2011)

- Reduction of the delay effect
- Accurate haptic rendering
Combining LKF with $H_\infty$

$$w(t) = \begin{pmatrix} F_e(t) \\ F_h(t) \end{pmatrix}$$

$$x(t) = \begin{pmatrix} \dot{\theta}_s(t) \\ \dot{\theta}_m(t) \\ \theta_s(t) - \theta_m(t) \end{pmatrix}$$

$$= z(t)$$

$$J(w) = \int_0^\infty (z(t)^T z(t) - \gamma^2 w(t)^T w(t))\, dt < 0$$

To ensure $J(w) < 0$, we consider the condition,

$$\dot{V}(t, x(t), \dot{x}(t)) + z(t)^T z(t) - \gamma^2 w(t)^T w(t) < 0$$

We choose a Lyapunov-Krasovskii Functional candidate,

$$V(t, x(t), \dot{x}(t)) = x(t)^T P x(t)$$

$$+ \int_t^{t-h_2} x(s)^T S_a x(s)\, ds + \int_t^{t-h_1} x(s)^T S x(s)\, ds$$

$$+ h_1 \int_{-h_1}^0 \int_{t+\theta}^{t} \dot{x}(s)^T R \dot{x}(s)\, ds\, d\theta$$

$$+ \sum_{i=1}^n (h_2 - h_1) \int_{-h_2}^0 \int_{t+\theta}^{t} \dot{x}(s)^T R_{ai} \dot{x}(s)\, ds\, d\theta$$

LKF for teleoperation (cont’d)
Tracking in abrupt changing motion:

- Left Figure: new proxy control scheme.
- Middle Figure: new control scheme (B. Zhang, A. Kruszewski, J.-P. Richard, CCDC, 2011).
- Right Figure: passivity theorem (Y. Ye, Y.-J. Pan and Y. Gupta, CDC, 2009).
Tracking in wall contact motion:

- The hard wall: the stiffness $K_e = 30kN/m$, the position $x = 1.0m$.
- Left Figure: new proxy control scheme.
- Middle Figure: new control scheme (B. Zhang, A. Kruszewski, J.-P. Richard, CCDC, 2011).
- Right Figure: passivity theorem (Y. Ye, Y.-J. Pan and Y. Gupta, CDC, 2009).

LAGIS ETFA’11
LAGIS CCDC’11
Passivity
the proxy structure allows for reducing the lag

and gives a good force rendering...
Intermediate conclusion on bilateral teleoperation

• Classical solution = passivity, without performance

• Alternative LFK + Hinf, various possible structures:
  
  – position/Position (CCDC 2011) \(\Rightarrow\) position tracking OK
  
  – force/Position (ETFA 2011) \(\Rightarrow\) position + haptic rendering
  
  – LMI conditions \(\Rightarrow\) stability + performance \(\gamma\)
  
  – under variable delays (estimated by means of time stamps+NTP)
  
  – and still *buffer-free*...
An other work on bilateral teleoperation

"Network-based haptic systems with time-delay"
B. LIACU 2012 (NICULESCU, ANDRIOT, COLLEDANI, BOUCHER, DUMUR)

- **Aim:** improve the end user *perception*.
- **Smith predictor with distance feedback**, uses the information from the virtual environment so to have a better predictor model in wall contact.
- Experiments with simulated constant delays 50-80ms.
Overview

• Motivations and examples
• Modelling
• Sampling and delay
• A very basic example
• Control: to buff, or not to buff? A selection of results
• Conclusions
Conclusion-Summary

- Quite general problems formulated thx to delay/switch models
  \[ \Rightarrow \textit{design in the presence of variable delays} \]

- Control with a network model [Witrant 05] predictor with variable delay

- General case (without network model):
  - with 2 buffers [Estrada 08] retarded error equation
  - with 1 buffer [Seuret 06] state feedback + observ./predict.
  - + adapt. to QoS [Jiang-Kruszewski-R-T 08]
  - with 0 buffer [S-R 08, Jiang-K-R-T 09]

- Bilateral teleoperation with performance (alternative to passivity)
  - with 0 buffer [Zhang-K-R 11] state feedback + obs + $H_\infty$
Other healthy readings:

"A switched system approach to exponential stabilization through communication network"
A. KRUSZEWSKI, W.J. JIANG, E. FRIDMAN, J.P. RICHARD, A.TOGUYENI

"A novel control design for delayed teleoperation based on delay-scheduled Lyapunov-Krasovskii functionals“
A. KRUSZEWSKI, B. ZHANG, J.P. RICHARD
Int. J. Control, accepted May 2013, to appear.

"Control design for teleoperation over unreliable networks: A predictor-based approach"
A. KRUSZEWSKI, B. ZHANG, J.P. RICHARD

"Remote Stabilization via Communication Networks with a Distributed Control Law"
E. WITRANT, C. CANUDAS DE WIT, D. GEORGES, M. ALAMIR

"Master-Slave synchronization for two inverted pendulums with communication time-delay“
H J. ESTRADA-GARCIA, L.A. MARQUEZ-MARTINEZ, C.H. MOOG

"Control of a remote system over network including delays and packet dropout"
A. SEURET, J.P. RICHARD
IFAC'08, 17th IFAC World Congress, Seoul, South Korea, July 2008

“A refined input delay approach to sampled-data control”
E. FRIDMAN, Automatica, 48 (2), 2010

“SOS for sampled-data systems“
A. SEURET, M. PEETS, IFAC’11, Milano, Italy, 2011

“A novel stability analysis of linear systems under asynchronous samplings”.
A. SEURET, Automatica 48 (1), pp. 177-182, 2011
Scheduling issues: aperiodic sampling for nonlinear systems (PhD Hassan Omran Feb. 2014 → hybrid/dissipativity)

"Stability of bilinear sampled-data systems with an emulation of static state feedback"
H. OMRAN, L. HETEL, J.P. RICHARD, F. LAMNABHI-LAGARRIGUE
CDC’12, 51st Conf. on Decision & Control, Hawaii, USA, 2012

"On the stability of input-affine nonlinear systems with sampled-data control"
H. OMRAN, L. HETEL, J.P. RICHARD, F. LAMNABHI-LAGARRIGUE
ECC’13, 12th European Control Conf., Zurich, Switzerland, 2013

Keep on developing experimental platforms

- NCS@LAGIS (various benchtests with Wifi – Zigbee – LAN – CAN – wired)
- EquipEx FUN Future Internet of Things (WSRN)