M2 internship project:
Weak metrics for gradient descent in seismic imaging by 1D full waveform inversion

▷ **Context:** Full waveform inversion (FWI) is a high resolution seismic imaging technique based on an iterative data fitting procedure (Virieux et al., 2017). The match between observed data and synthetic data computed through the numerical solution of partial differential equations is iteratively improved following local optimization methods. Since 2010 and a spectacular application on 3D field data from the North sea (Sirgue et al., 2010), FWI has become a standard velocity building tool in the exploration industry. It has been also adopted for imaging the Earth's structure at various scales, from global-regional tomography, to near surface characterization for the detection/monitoring of fluid contents.

It has been understood in the recent years (see e.g. Métivier et al. (2019)) that the use of distances arising from optimal transport theory to evaluate the misfit between observed and synthetic data greatly mitigates the non-convexity of the optimization problem to be solved to fit the model. In the framework of the MathSout project of the PEPR Maths-Vives (`https://www.maths-vives.fr/projet/mathsout/`), we want to further explore the influence of the choice of the metric, but in the model space rather than in the data space.

▷ **Project:** In mathematical terms, the FWI amounts to find the best scalar (for simplicity) field $m : \mathbb{R}^d \supset \Omega \to \mathbb{R}$ – typically the velocity of the wave propagation as a function of space – minimizing the misfit between the observed reflected signal d_{obs} at the receiver position with the one $d_{cal}[m]$ corresponding to the evolution of waves in a medium corresponding to the field m, i.e.

$$\min_m f(m), \quad f(m) = \frac{1}{2}\|d_{cal}[m] - d_{obs}\|^2, \quad A(m)u = b, \quad d_{cal}[m] = Ru[m]. \tag{1}$$

In the above problem, $f(m)$ is the FWI misfit function, $d_{cal}[m] = Ru[m]$ is the restriction of the wavefield $u[m]$ at the receiver position through the extraction operator R, with $u[m]$ being the solution of a wave equation with parameter m, denoted for short $A(m)u = b$ where b is the seismic forcing.

A natural approach to solve the minimization problem (1) is to use a gradient descent strategy and, starting from an initial guess m_0, to solve

$$m_{k+1} + \nabla_m f(m_{k+1}) = m_k, \qquad k \geq 0. \tag{2}$$

In (2), $\nabla_m f$ stands for the gradient (w.r.t. m) of the misfit function f, which relates to the differential of f through the choice of a scalar product on a Hilbert space H, i.e.

$$\langle \nabla_m f(m), h \rangle_H = Df(m) \cdot h = \int_\Omega f'(m)h, \qquad \forall h \in H,$$

with $f'(m)$ being the Fréchet derivative of f at m. While the canonical scalar product on $H = L^2(\Omega)$ is the default choice in the numerical codes, we aim to investigate the influence of the scalar product H on the efficiency of the gradient descent algorithm (2).

A first natural choice is to use weighted $(H^1(\Omega))^{-1}$ scalar product, so that $\mu = \nabla_m f(m)$ solves the elliptic equation

$$-\ell^2 \Delta\mu + \mu = f'(m) \quad \text{in } \Omega, \qquad \text{with} \quad \nabla\mu \cdot n = 0 \quad \text{on } \partial\Omega. \tag{3}$$

In (3), the gradient and Laplace operators relate to derivatives with respect to the space variable $x \in \Omega$. The positive parameter ℓ is a characteristic distance to be adapted in order to improve the behavior of the gradient descent (2).

A second more involved choice will consists in considering a Riemannian manifold $\mathcal{M}$ rather than a linear space for the model space, and to allow the scalar product on the tangent space $H = T_m\mathcal{M}$ to $\mathcal{M}$ at m to depend on m itself, in close connection with Otto's calculus (Otto, 2001) and (unbalanced) optimal transportation (Liero et al., 2018).

The aim of the internship is to provide a preliminary analysis of this problem, supported by numerical results where we will consider a 1D acoustic wave propagation problem. Depending on the advancement of the project, a continuation towards a PhD training is possible (secured funding by from PEPR Maths-Vives).

▷ **Contacts:**

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▷ **Collaborations:** The intern will be hosted in the Rapsodi team (https://team.inria.fr/rapsodi/) in the Inria Center at Univ. Lille under the supervision of Clément Cancès, and with a strong collaboration with Ludovic Métivier and the SEISCOPE team (https://seiscope2.osug.fr) within the ISTerre laboratory (https://www.isterre.fr/) in Univ. Grenoble Alpes.

▷ **Competences:** Partial differential equations, Optimization, Numerical analysis, Scientific computing (Python, Matlab, FORTRAN90).

▷ **Duration:** up to 6 months, starting from April 2026 or later

▷ **Grant:** About 640€ per month

▷ **Application:** https://recrutement.inria.fr/public/classic/en/offres/2025-09603

References

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Métivier, L., Brossier, R., Mérigot, Q., and Oudet, E. (2019). A graph space optimal transport distance as a generalization of L^p distances: application to a seismic imaging inverse problem. *Inverse Problems*, 35(8):085001.

Otto, F. (2001). The geometry of dissipative evolution equations: the porous medium equation. *Comm. Partial Differential Equations*, 26(1-2):101–174.

Sirgue, L., Barkved, O. I., Dellinger, J., Etgen, J., Albertin, U., and Kommedal, J. H. (2010). Full waveform inversion: the next leap forward in imaging at Valhall. *First Break*, 28:65–70.

Virieux, J., Asnaashari, A., Brossier, R., Métivier, L., Ribodetti, A., and Zhou, W. (2017). An introduction to Full Waveform Inversion. In Grechka, V. and Wapenaar, K., editors, *Encyclopedia of Exploration Geophysics*, pages R1–1–R1–40. Society of Exploration Geophysics.