Beyond Admissibility : Dominance between chains of strategies

Marie van den Bogaard[1]

Joint work with N. Basset[2], I. Jecker[1], A.Pauly[3] & J.-F. Raskin[1]

Presented at CSL 2018

[1]Université Libre de Bruxelles, [2]Université Grenoble Alpes, [3]Swansea University

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What does it mean to act *rationally* in an interactive scenario?

(especially when there is no obvious optimal choice?)

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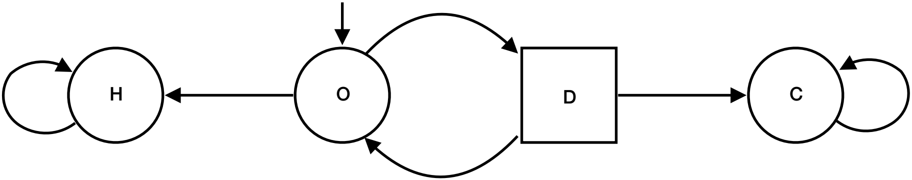
Let's play. . .

With the point of view of the first (circle) player

Let's play...

With the point of view of the first (circle) player



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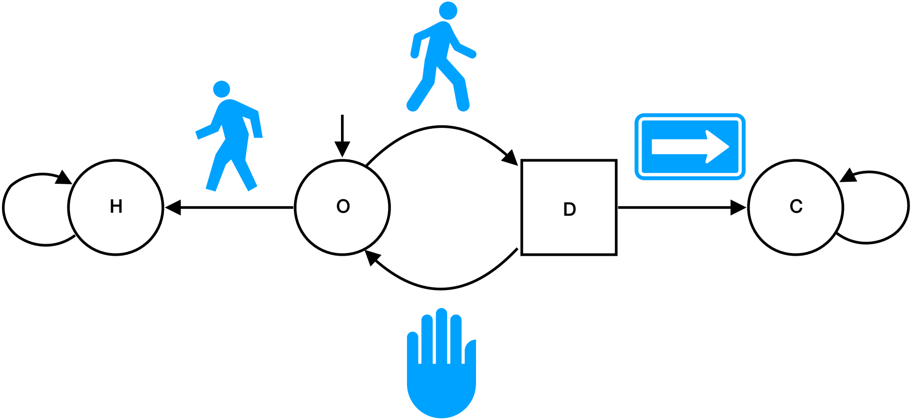

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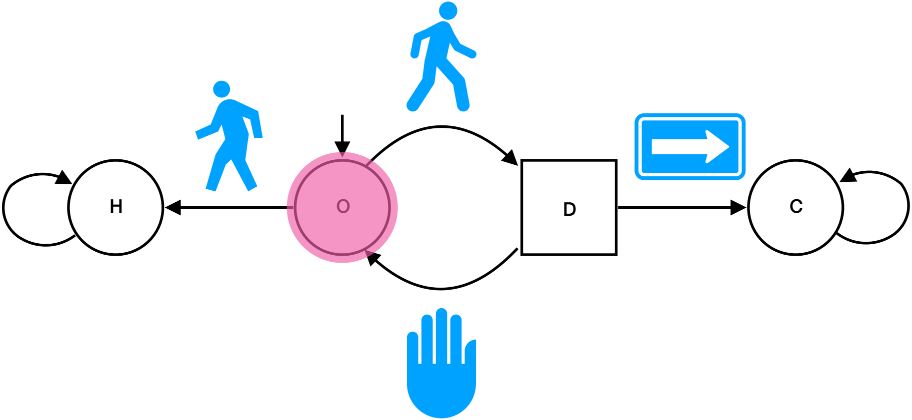

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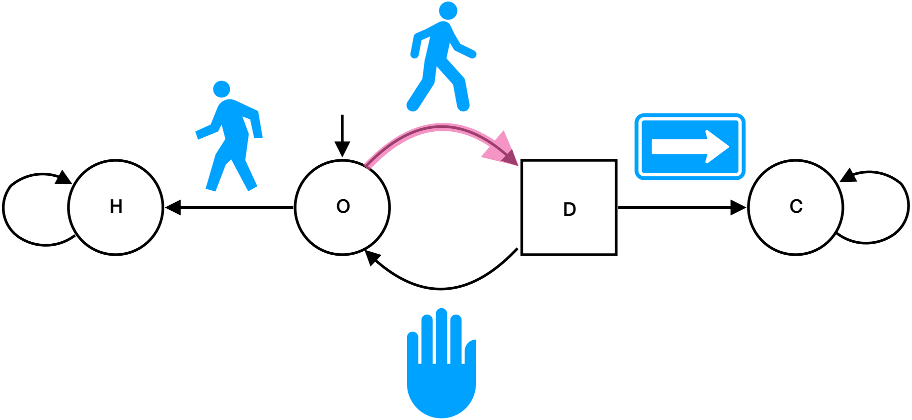

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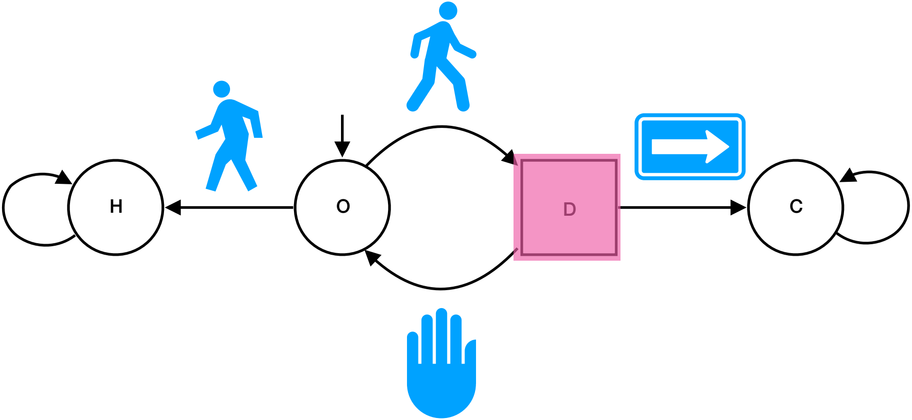

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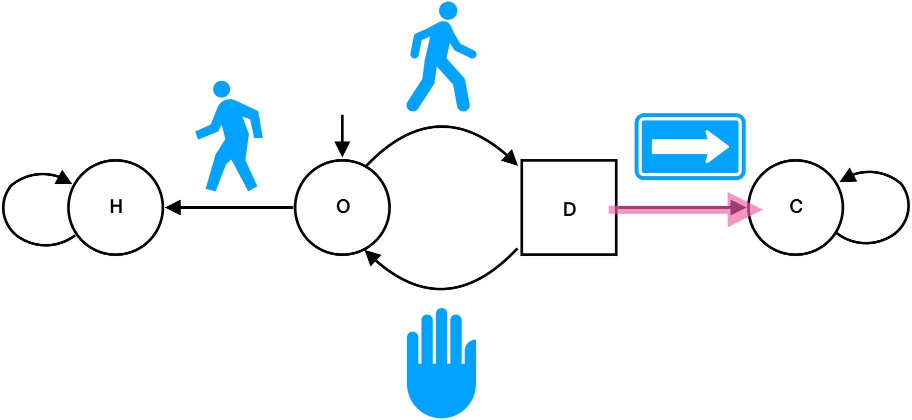

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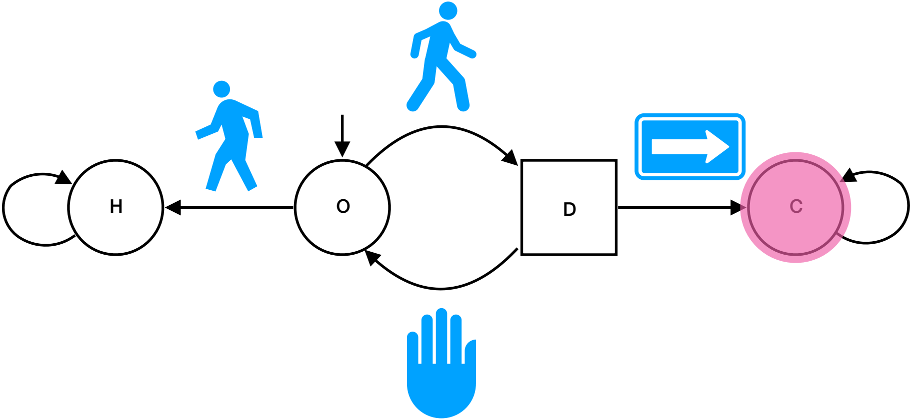

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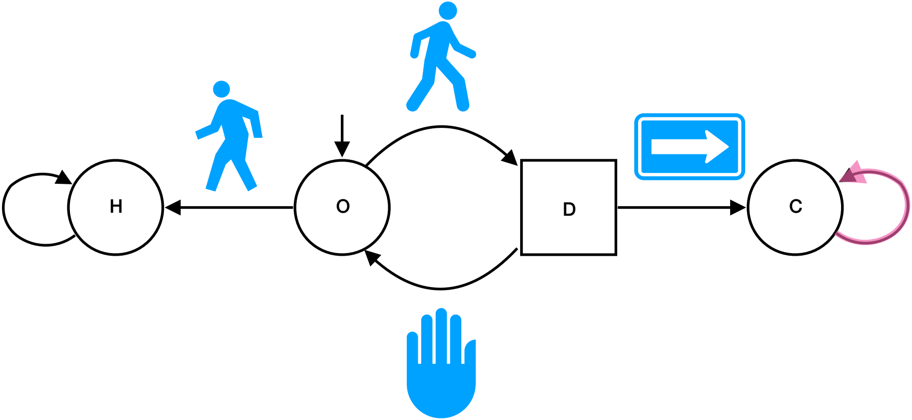

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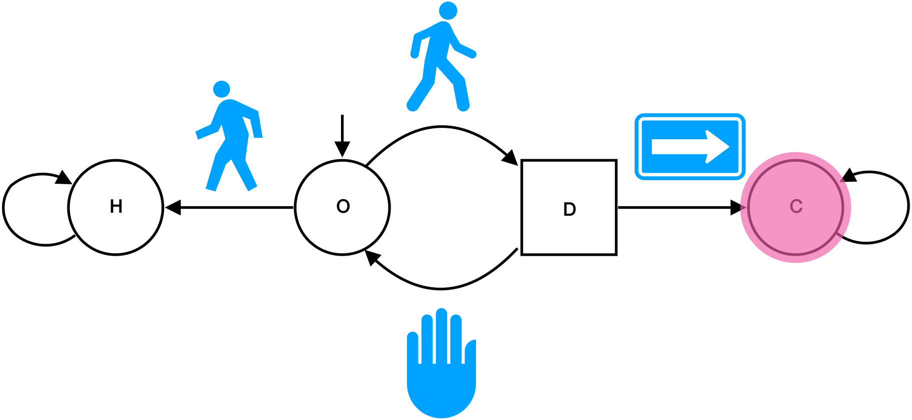

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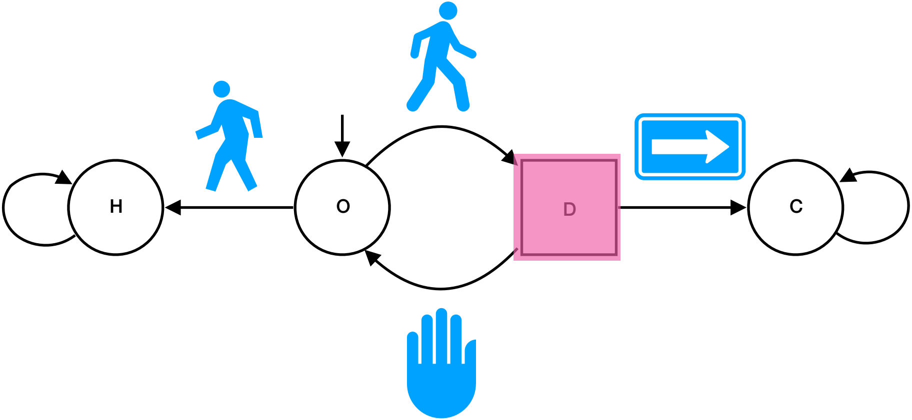

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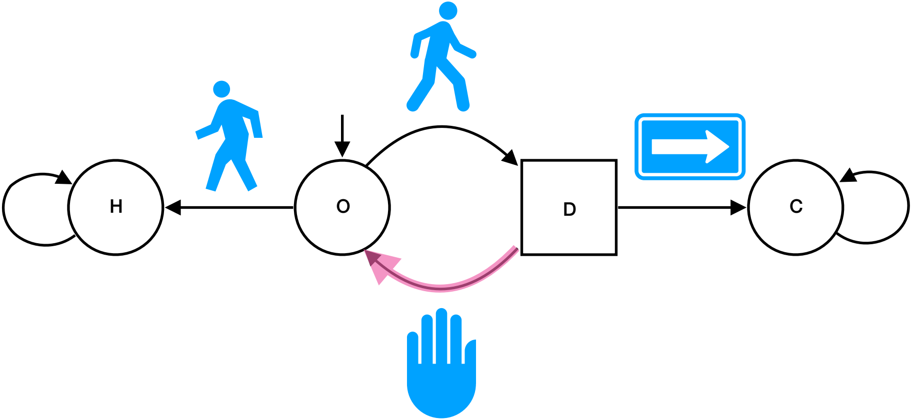

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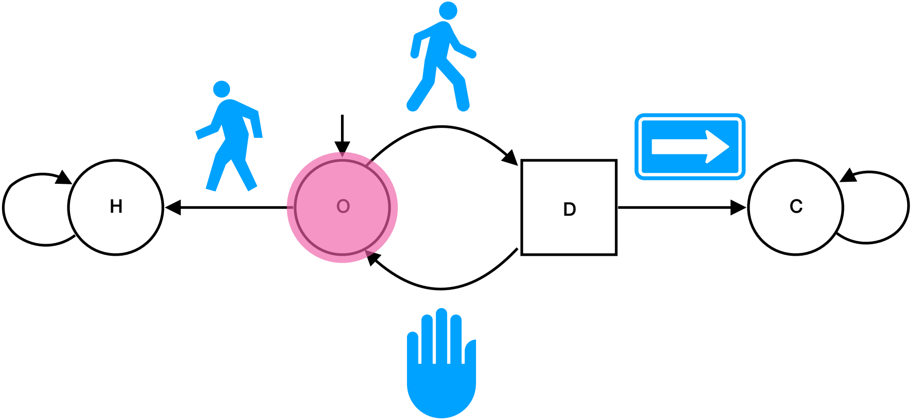

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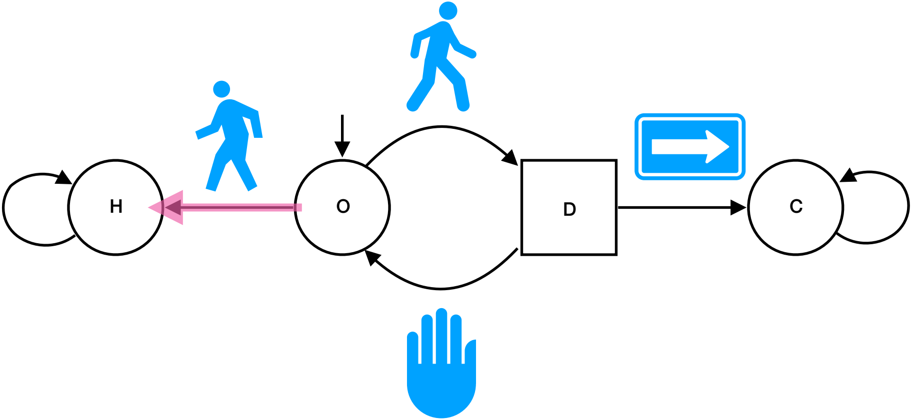

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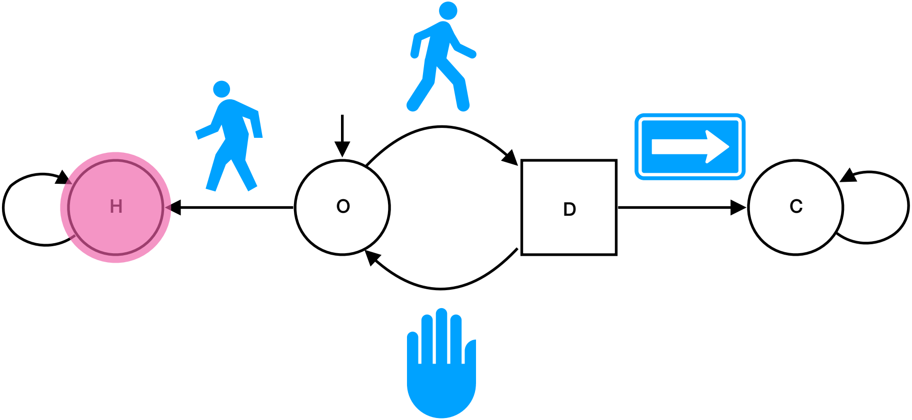

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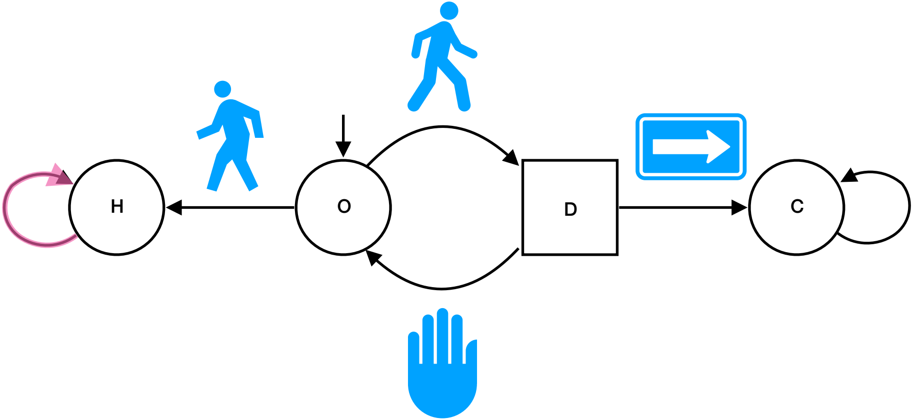

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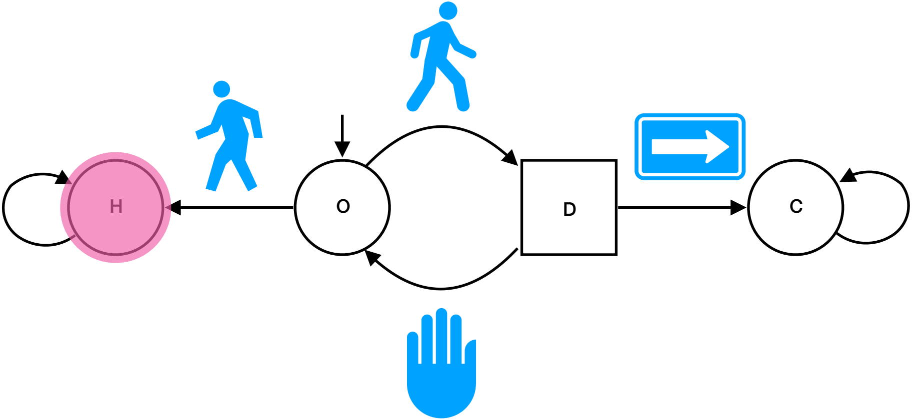

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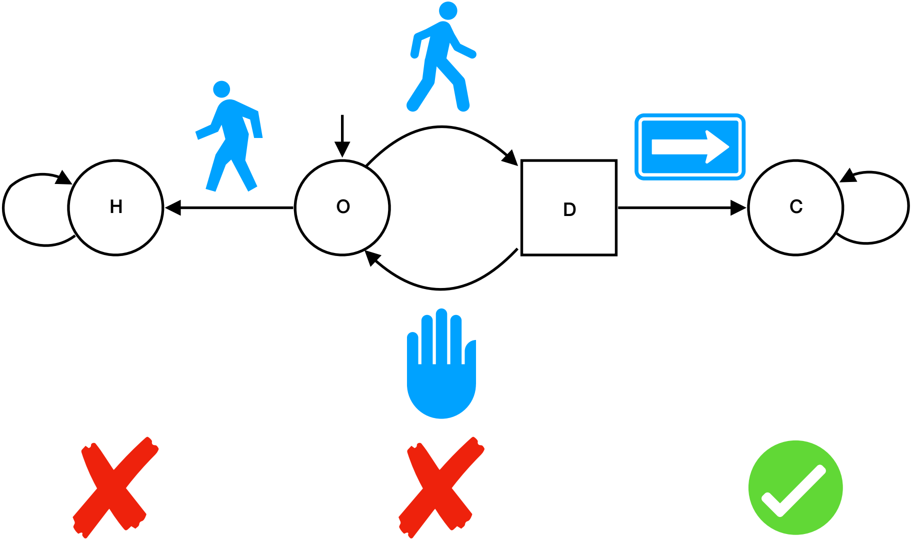

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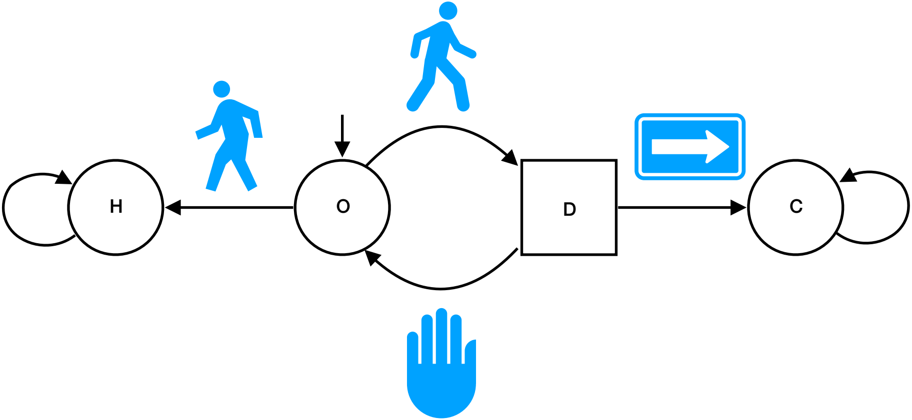

Is there a strategy that lets me enter regardless of the doorman actions?

Let's play...

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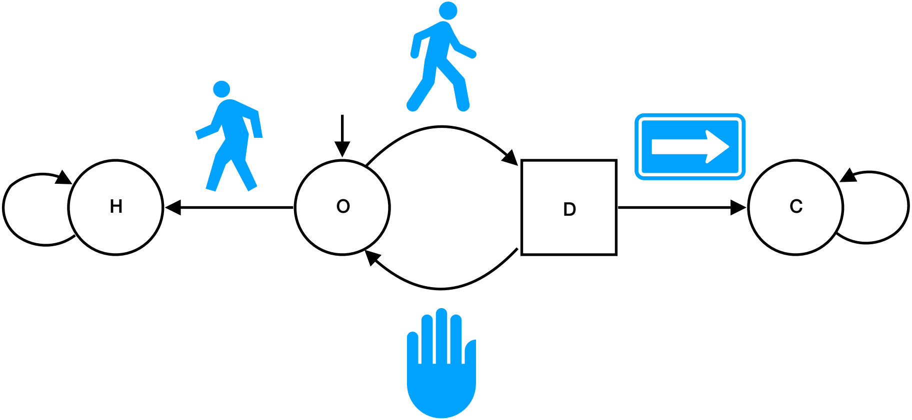
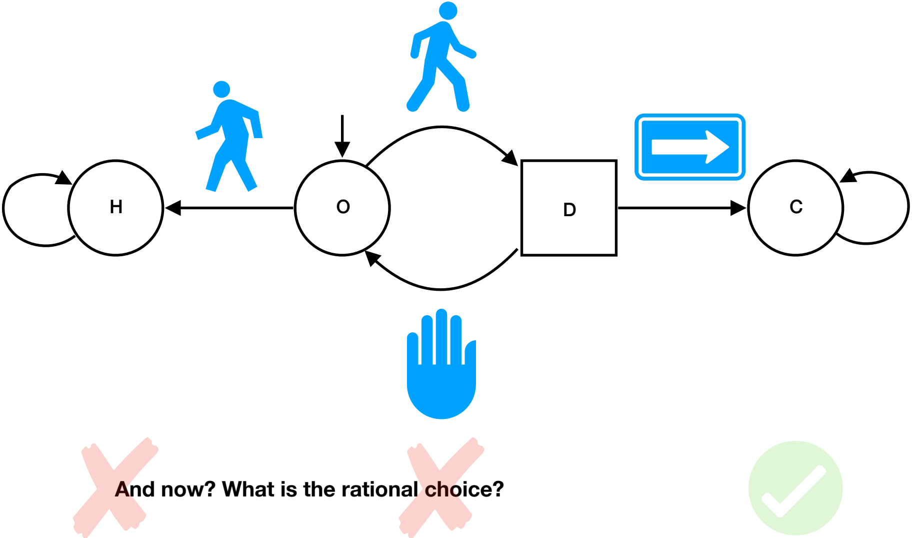


Is there a strategy that lets me enter regardless of the doorman actions?

NO.

Let's play...

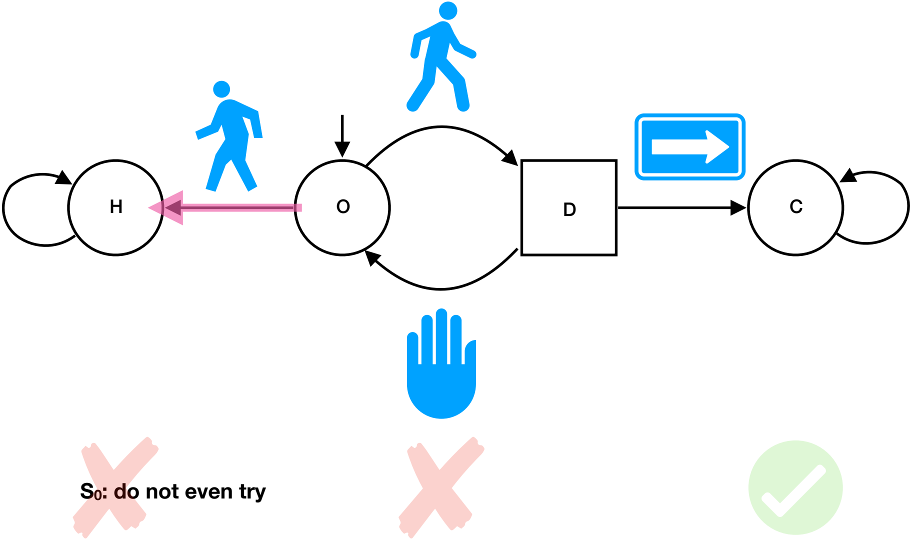
With the point of view of the first (circle) player



And now? What is the rational choice?

Let's play...

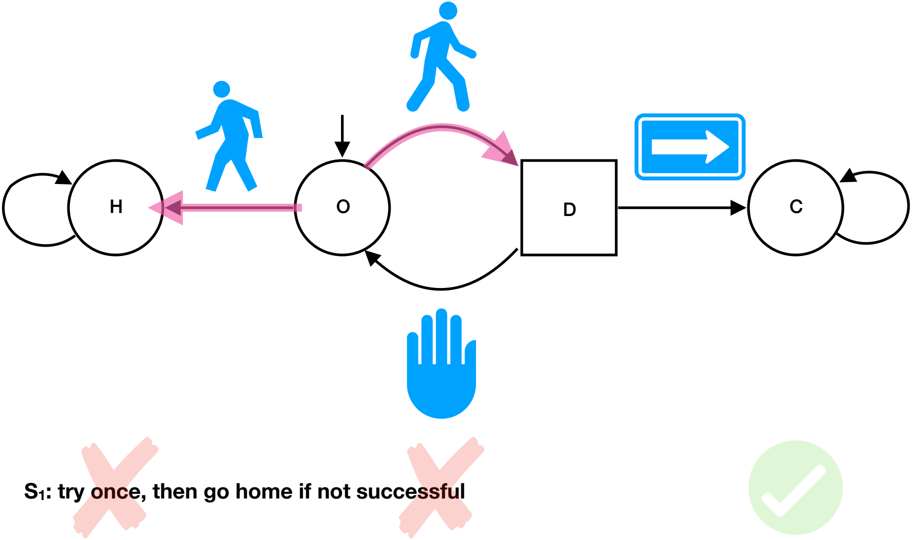
With the point of view of the first (circle) player



S_0: do not even try

Let's play...

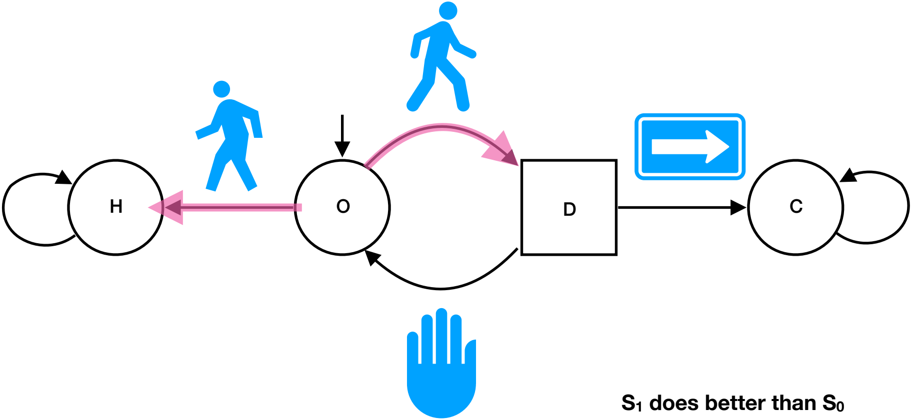
With the point of view of the first (circle) player



S_1: try once, then go home if not successful

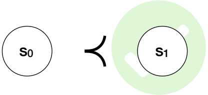
Let's play...

With the point of view of the first (circle) player



S_1: try once, then go home if not successful

S_1 does better than S_0
S_1 *dominates* S_0



Let's play. . .

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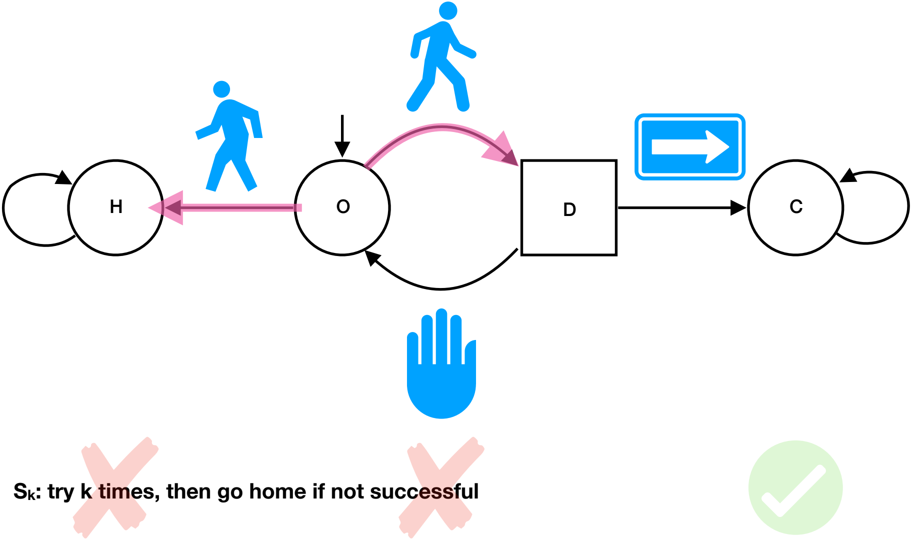
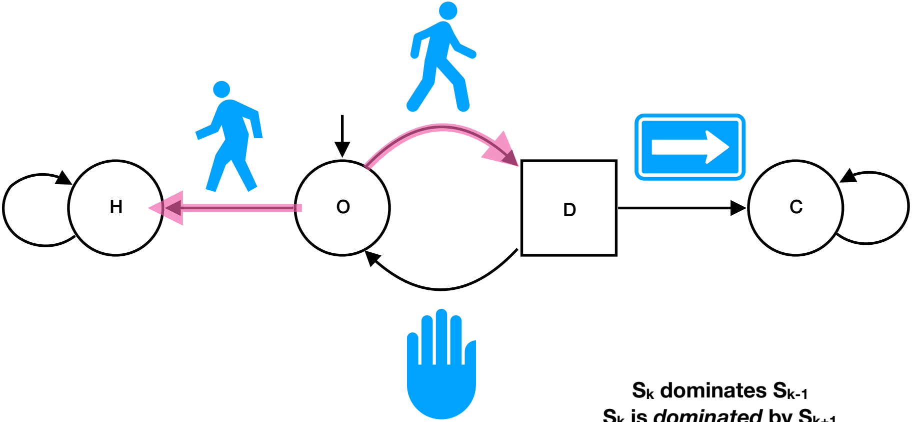


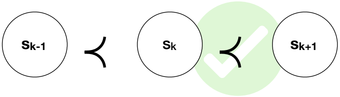
S_k: try k times, then go home if not successful

Let's play. . .

With the point of view of the first (circle) player



S_k dominates S_{k-1}
S_k is *dominated* by S_{k+1}



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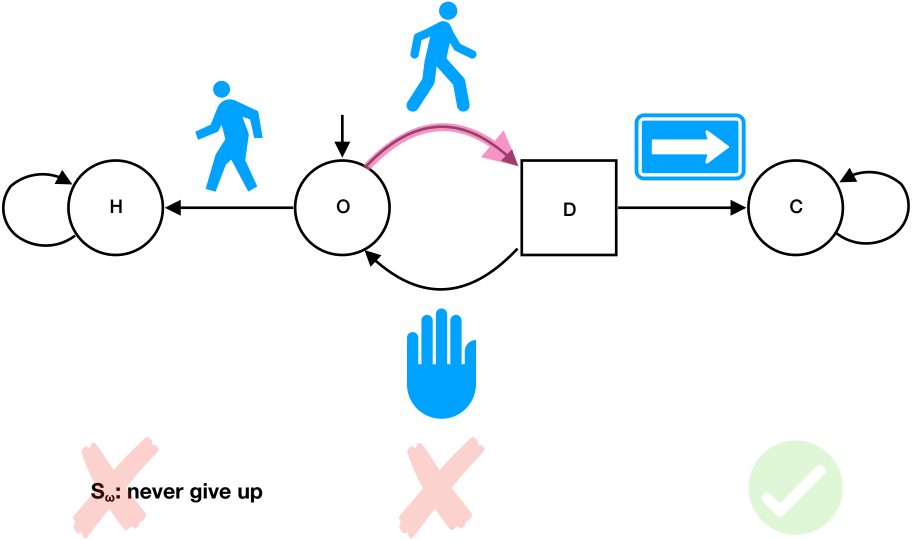

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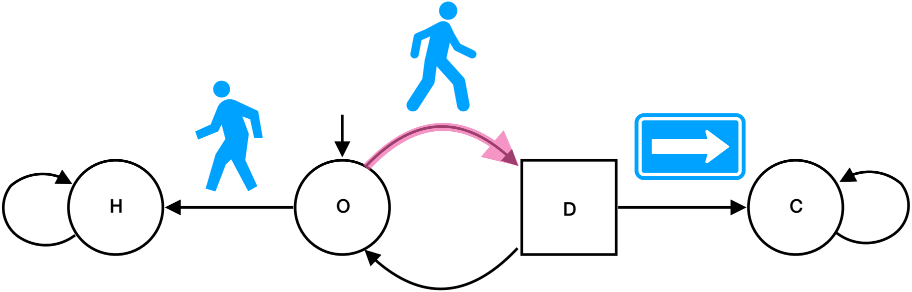

S_ω dominates every S_k

&

S_ω is *not* dominated by any S_k

S_ω is *admissible*

S_ω: never give up

$S_k \prec S_\omega$ & $S_\omega \not\prec S_k$

Dominance

$$s \prec s'$$

A strategy s *is dominated* by a strategy s' (or s' *dominates* s) if :

(*a*) for every strategy (profile) τ of the other player(s) :

$$p(s,\tau) \leq p(s',\tau)$$ "s' is always as good as s"

(*b*) there exists a strategy (profile) τ of the other player(s) such that

$$p(s,\tau) < p(s',\tau)$$ "s' sometimes better than s"

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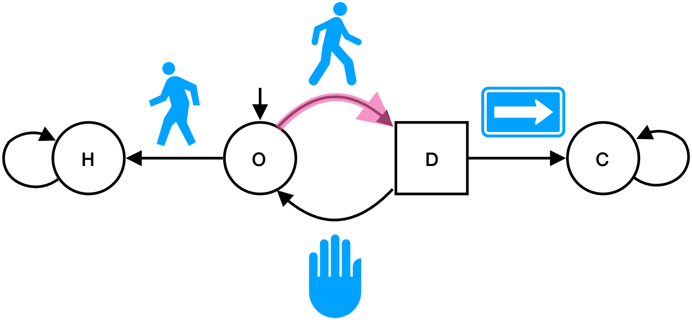
$$p(s, \tau) < p(s', \tau)$$ "s' sometimes better than s"

If only (a) holds, then $s \preceq s'$: strategy s' *weakly* dominates strategy s.

Admissibility

A strategy s is *admissible* if it is **not** dominated by any other strategy :

$$\text{for every } s', \text{ we have } s \not\prec s'.$$



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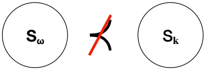

Let's play again . . .

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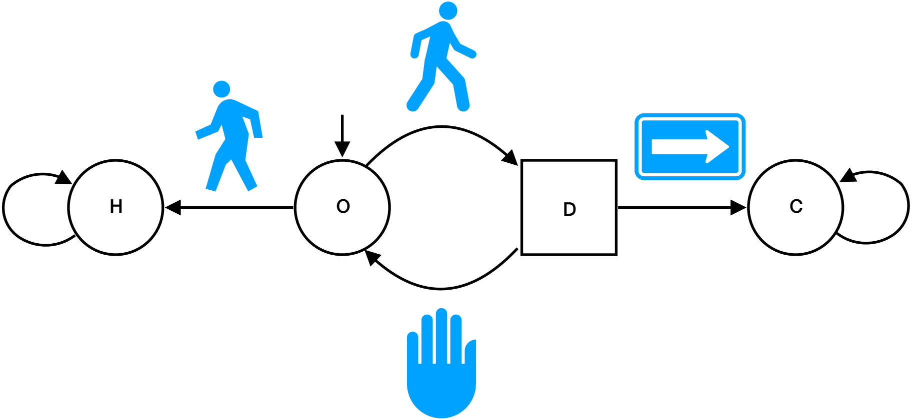

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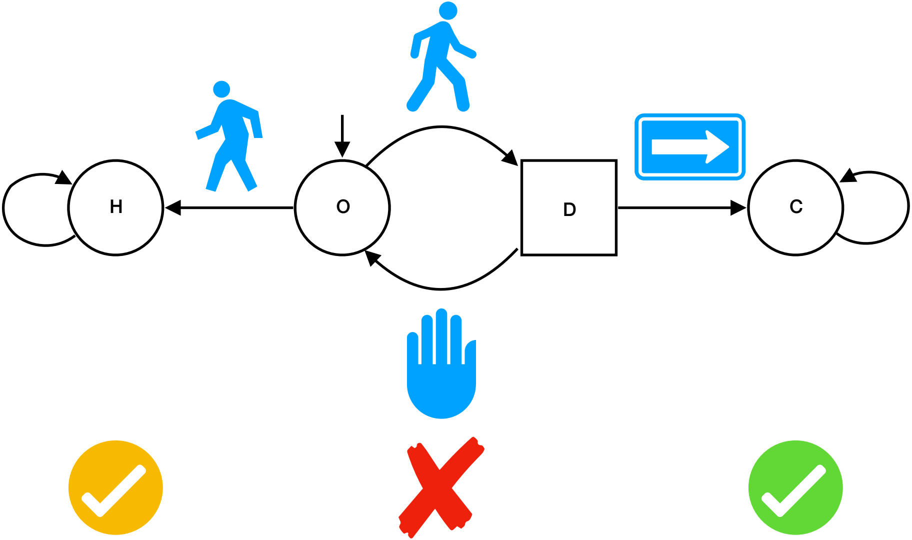
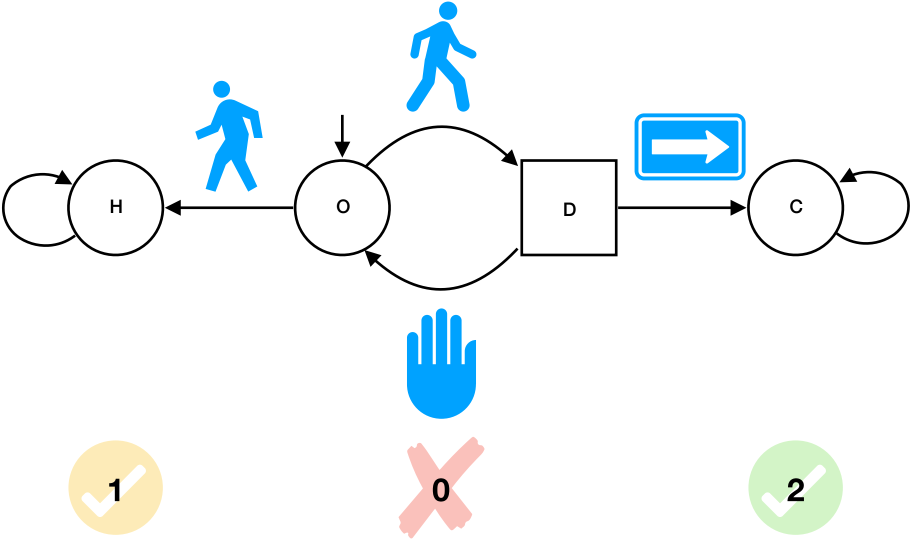


Let's play again ...



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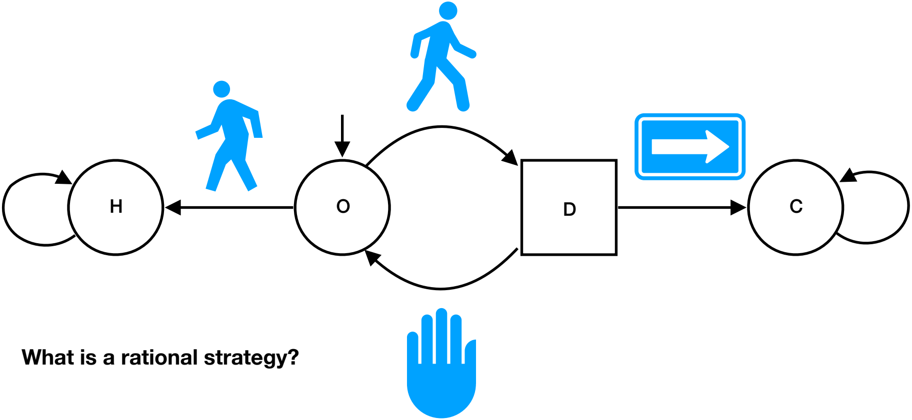
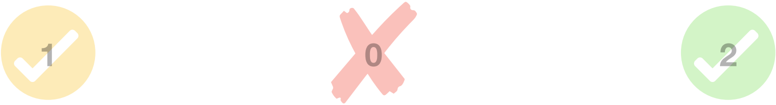


What is a rational strategy?



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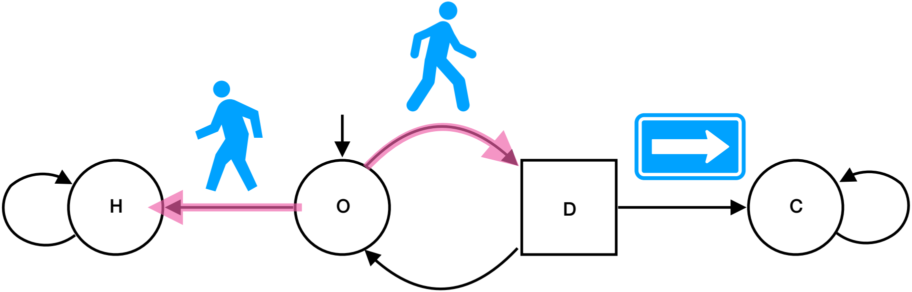

What is a rational strategy?

S_k dominates S_{k-1}
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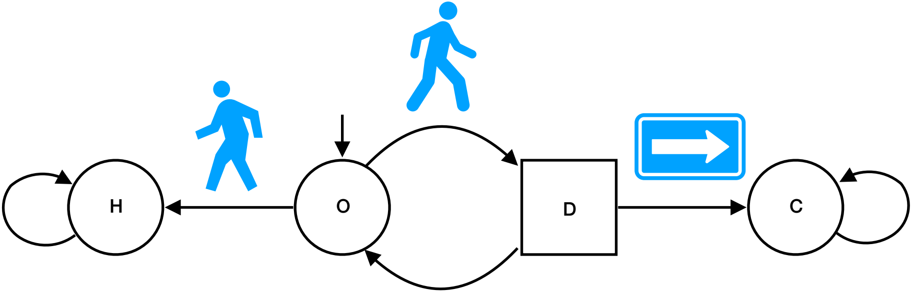
S_k: try k times, then go home if not successful

Let's play again . . .



What is a rational strategy?

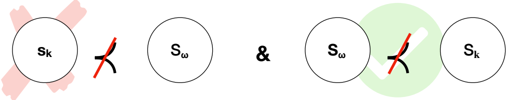
S_ω is *not* dominated by any S_k

(*admissible*)

but

S_ω does not dominate any S_k

S_ω: never give up



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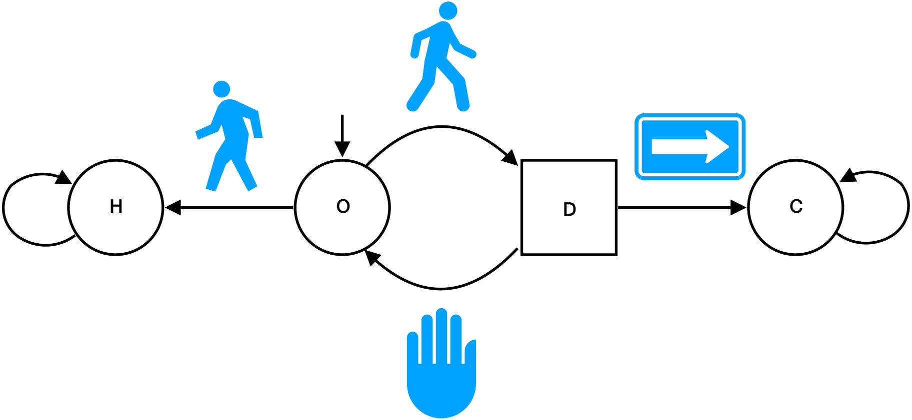
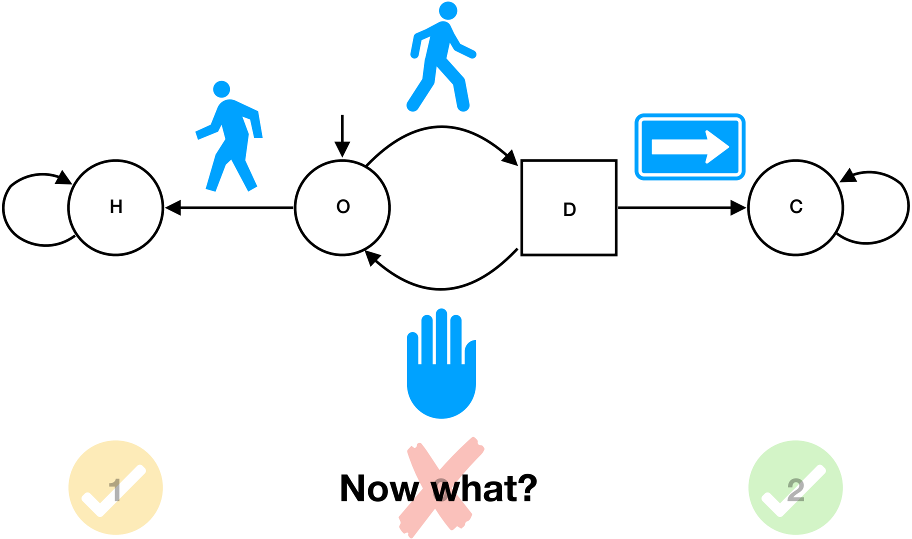


S_ω and S_k incomparable w.r.t. dominance

S_ω is not the only rational choice !

admissibility criterion is too restrictive

Let's play again . . .



Some formalities

Model : $\mathcal{G} = \langle P, G, (p_i)_{i \in P} \rangle$

- multiplayer turn-based games on finite graphs
- game graph : $G = (V = \uplus_{i \in P} V_i, E)$
- Player i strategies : $\Sigma_i = \{s : V^* V_i \to V\}$ *(from histories to vertices)*
- payoff functions : $p_i : V^\omega \to \mathbb{R}$ *(from outcomes to reals)*

Key points :

- focus on one player point of view
- no "adversarial opponent" hypothesis :
  - $\rightsquigarrow$ no assumptions about the other player(s) objectives / preferences

Boolean case

Admissibility is a good criterion of rationality in the boolean case :

- always exist for ω-regular winning objectives
- admissible strategies coincide with winning strategies (when these exist)

- every strategy is either :

  admissible or dominated by an admissible strategy

Fundamental property!

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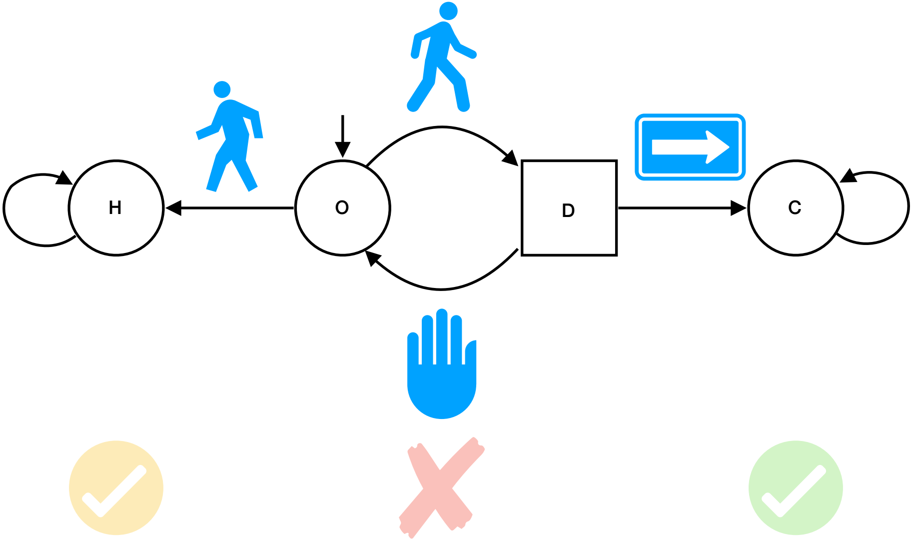
Fundamental property!

Bonus : iteration, synthesis . . .

[Berwanger '07, Faella '09, Raskin et al.$^{+}$]

Quantitative case



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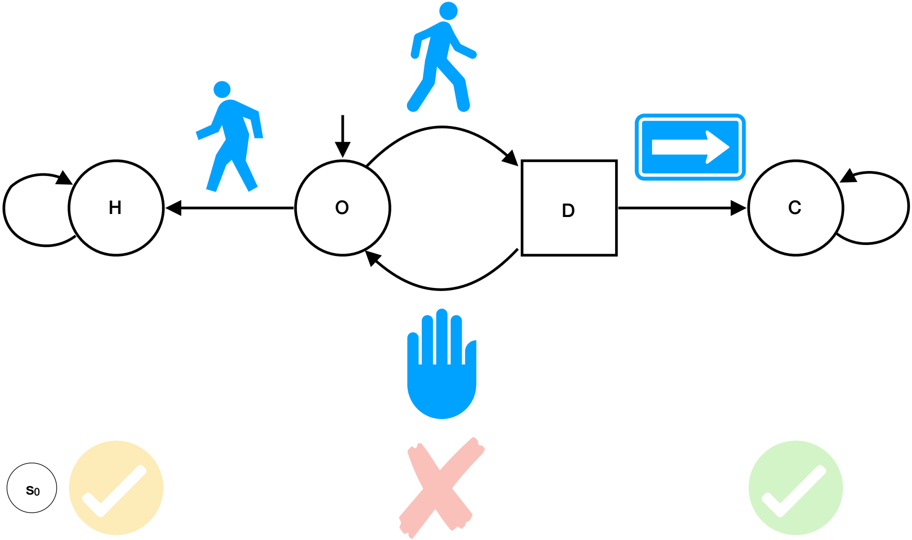

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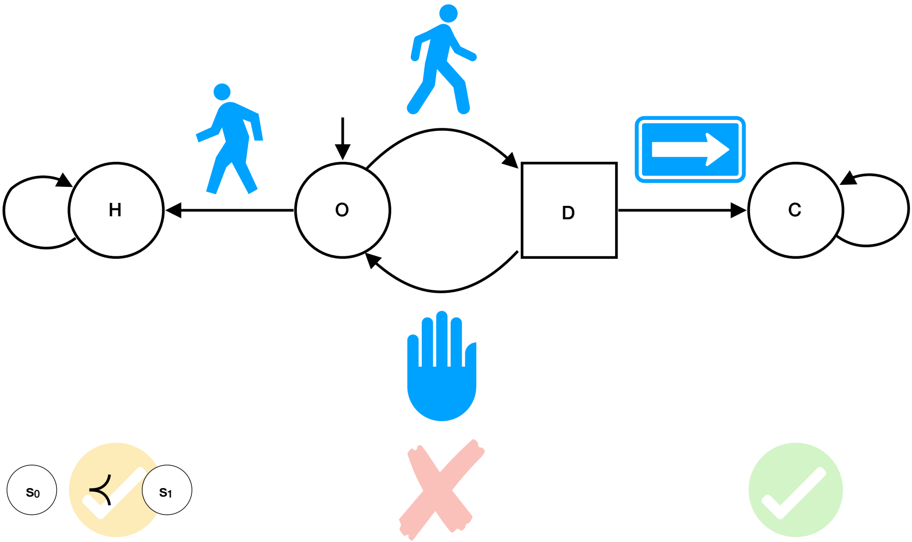

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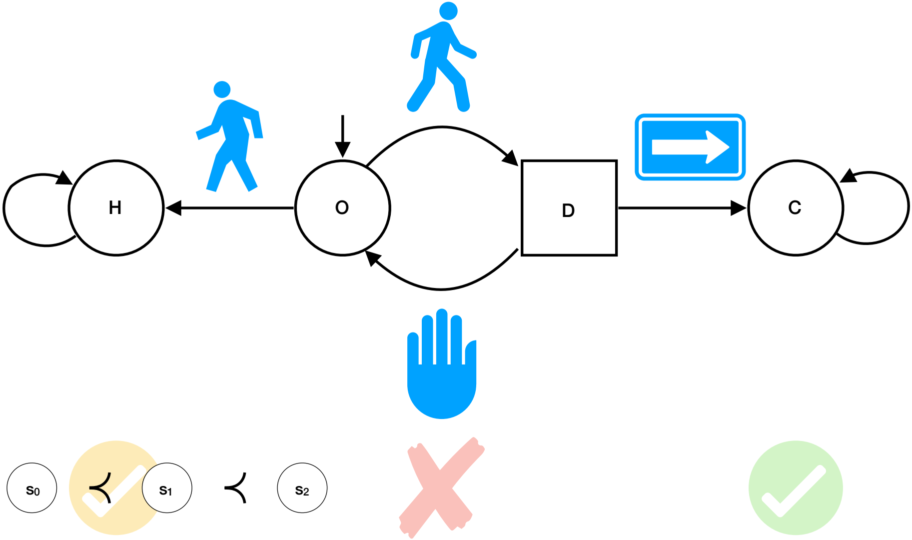

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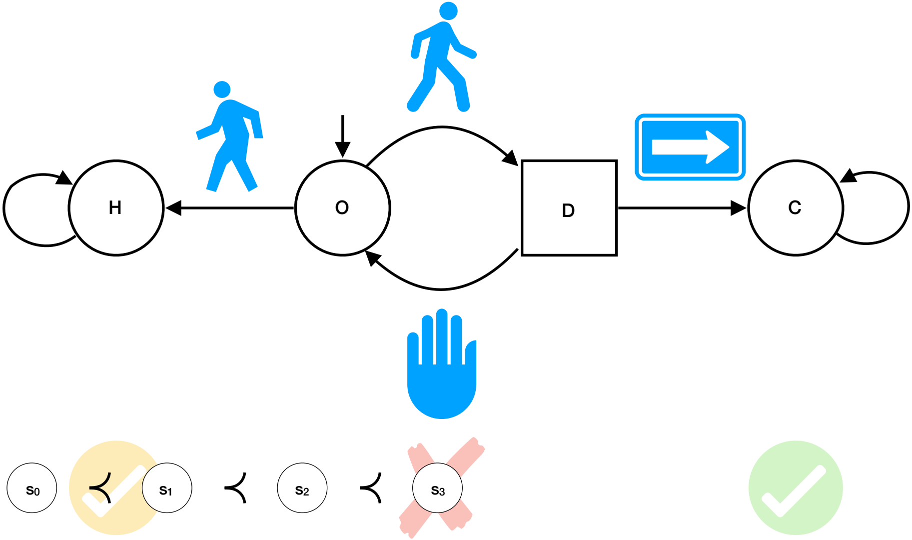

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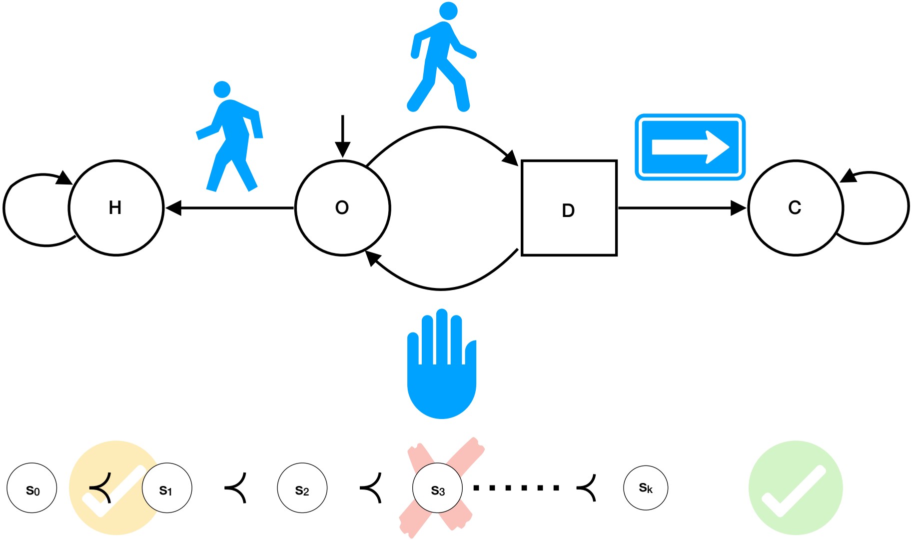

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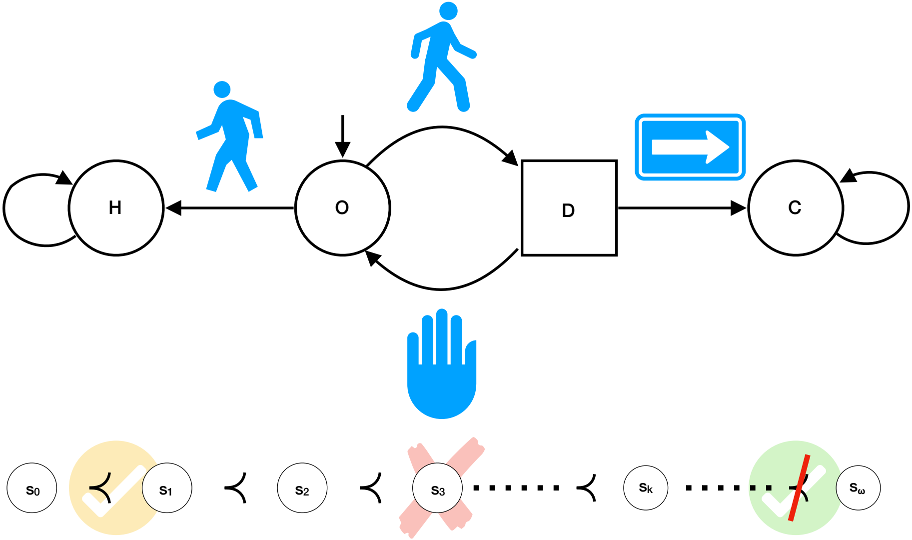


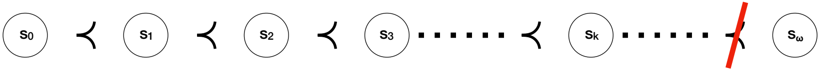
Quantitative case

As soon as there are 3+ payoffs :

Admissible strategies represent rational choices but . . .

- they do not always exist,
- even when they do, they do not cover all rational behaviours,
- no guarantee to satisfy the fundamental property :

  dominated strategies not dominated by an admissible strategy exist



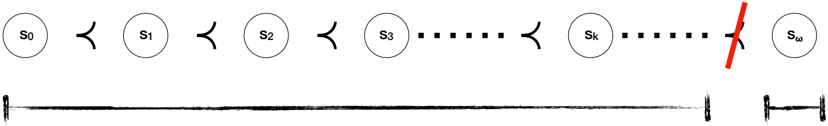
[Brenguier, Perez, Raskin, Sankur FSTTCS'16]

Quantitative case

New approach :

shift from singleton strategy analysis to consider families of strategies

Idea : cover rational behaviours dismissed by admissibility criterion



Quantitative case

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shift from singleton strategy analysis to consider families of strategies

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Chains **of strategies** **(sequences of strategies ordered by dominance)**

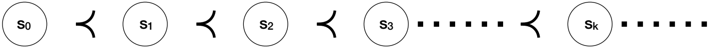
$$s_0 \prec s_1 \prec s_2 \prec s_3 \cdots\cdots \prec s_k \cdots\cdots \not\prec s_\omega$$

Chains of strategies in $(\Sigma_i, \preceq)$

A *chain of strategies* $(s_\alpha)_{\alpha<\beta}$ is a sequence of strategies, indexed by an ordinal $\beta > 0$, that respects the dominance quasiorder :

$$\text{for every } \alpha, \alpha' < \beta \text{ such that } \alpha < \alpha', \text{ we have } s_\alpha \preceq s_{\alpha'}.$$

Increasing chain : $(s_\alpha)_{\alpha<\beta}$ such that $s_\alpha \prec s_{\alpha'}$ for every $\alpha < \alpha'$.



Dominance between chains :

A chain $(s_\alpha)_{\alpha<\beta}$ is weakly dominated by a chain $(s'_{\alpha'})_{\alpha'<\beta'}$ if :
for every $\alpha < \beta$, there exists $\alpha' < \beta'$ such that $s_\alpha \preceq s_{\alpha'}$.

$$(s_\alpha)_{\alpha<\beta} \sqsubseteq (s'_{\alpha'})_{\alpha'<\beta'}$$

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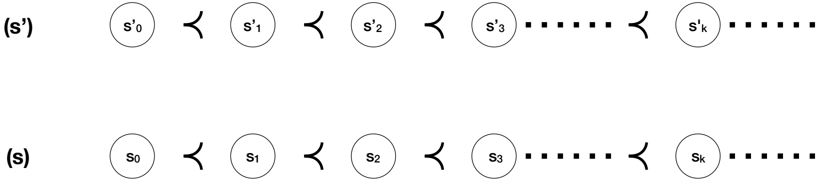

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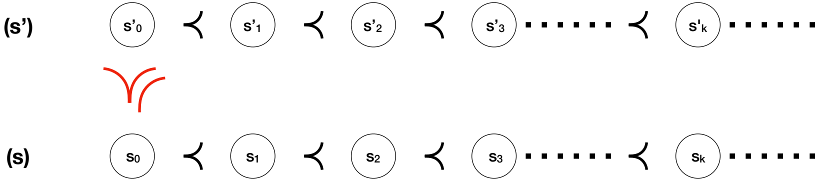

Dominance between chains :

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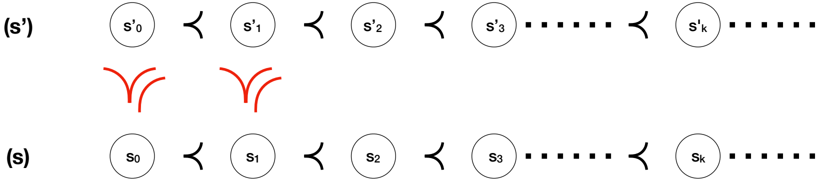

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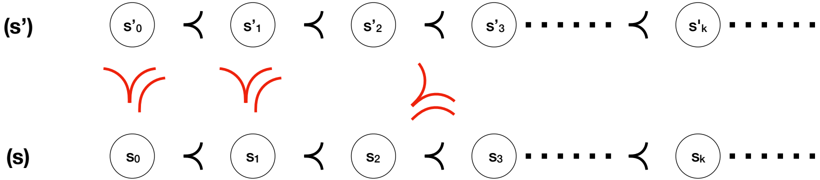

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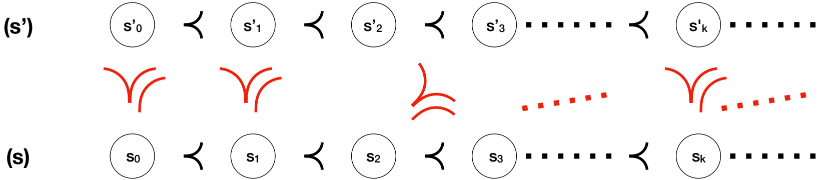

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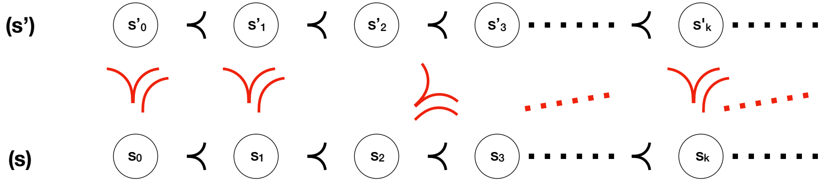

A chain $(s_\alpha)_{\alpha<\beta}$ is *maximal* if for every chain $(s'_{\alpha'})_{\alpha'<\beta'}$, we have

$$(s_\alpha)_{\alpha<\beta} \sqsubseteq (s'_{\alpha'})_{\alpha'<\beta'} \;\Rightarrow\; (s'_{\alpha'})_{\alpha'<\beta'} \sqsubseteq (s_\alpha)_{\alpha<\beta}$$

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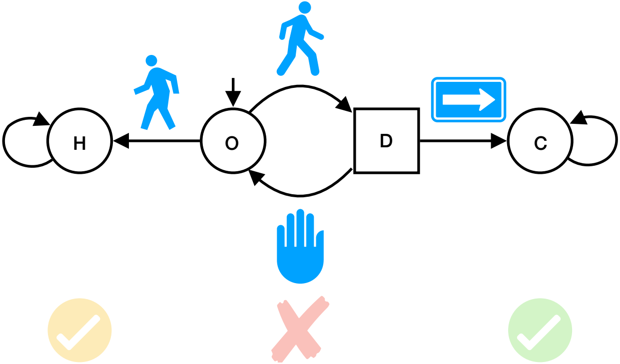
Chains of chains !

Considering $(IC(\Sigma_i), \sqsubseteq)$:
increasing chains of strategies and
the quasi-order $\sqsubseteq$,
we can build *chains of chains of strategies* :



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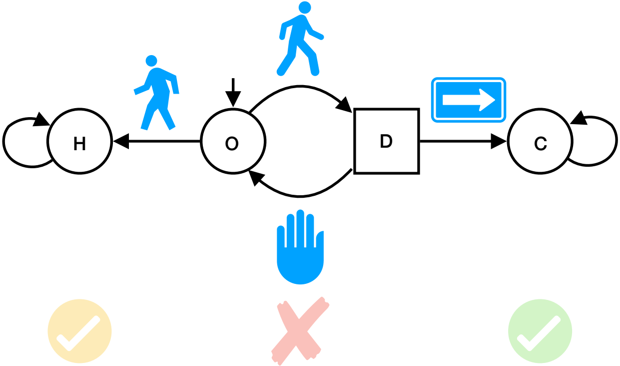

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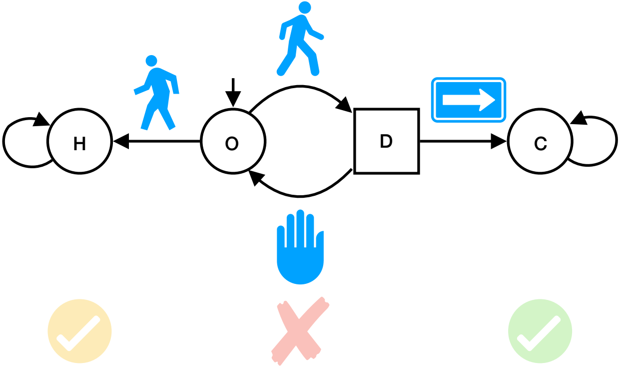

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A sufficient criterion...

...to recover a similar fundamental property as in the boolean case :

Theorem

If the chains of chains of strategies have at most a countable number of elements (chains of strategies), then every chain of strategies is either maximal or dominated by a maximal chain of strategies.

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Some proof ingredients :

(*i*) every increasing chain has countable length

(*ii*) every increasing chain of increasing chains has an upper bound

(*iii*) Zorn's Lemma !

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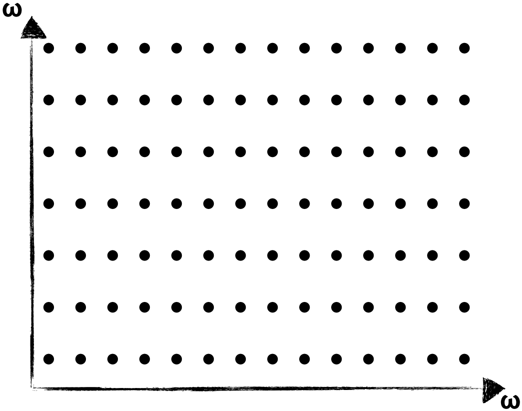

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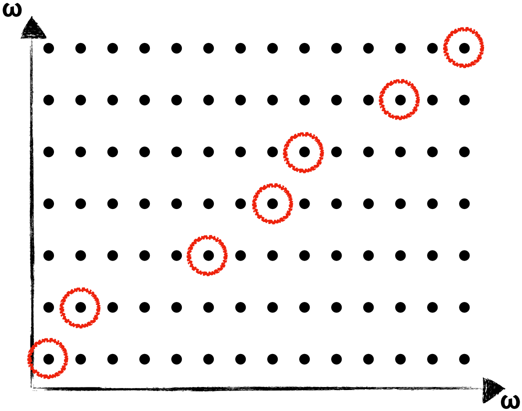

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(*iii*) **Zorn's Lemma :** Let $(X, \triangleleft)$ be a (quasi-)ordered set. If every chain in $(X, \triangleleft)$ has an upper bound, then every element of X is below a maximal element.

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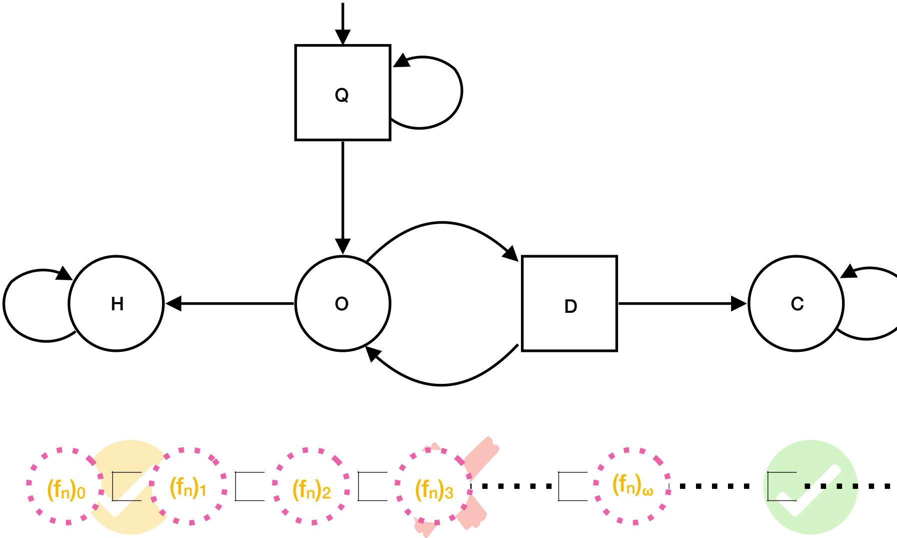

Length ω_1!

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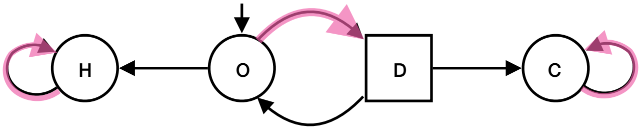
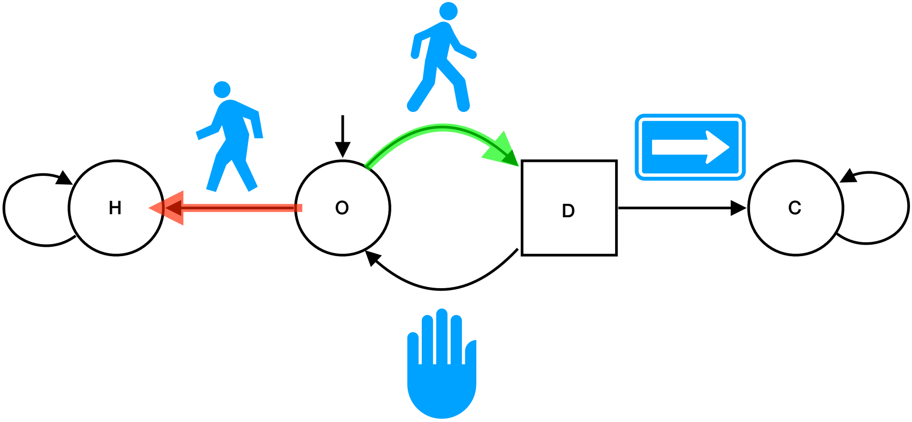


Parameterized automata to handle chains of strategies

Parameterized automaton : Mealy automaton with a single counter

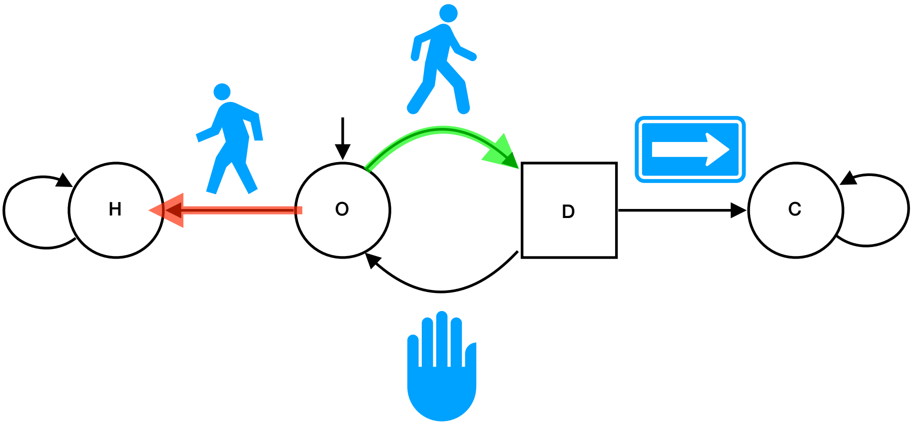
$\rightsquigarrow$ in counter-access states :

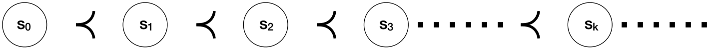
transition depends on the counter-value being > 0 or $= 0$.



Parameterized automata to handle chains of strategies

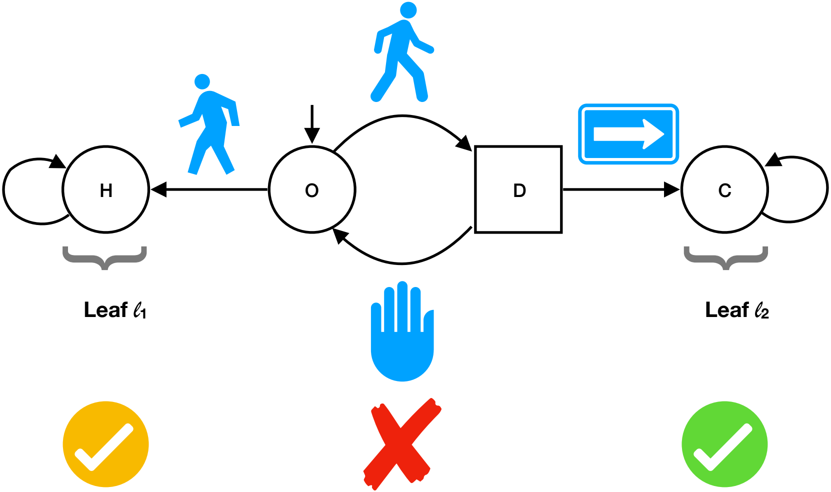
A chain is *uniform* if it is realized by a parameterized automaton





Generalised safety/reachability games

Games equipped with a set of leaves such that ending in leaf ℓ_n yields payoff n ($\in \mathbb{Z}$), while avoiding them yields payoff 0.



Parameterized automata to handle chains of strategies

In generalised safety/reachability games, considering finite-memory strategies :

- every dominated f.-m. strategy is dominated by an admissible f.-m. strategy or by a maximal uniform chain
- given a parameterized automaton, it is decidable whether it realizes an (increasing) chain
- dominance between two strategies is decidable
- dominance between two uniform chains is decidable

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