

AIL'17 Aide mémoire : Logique propositionnelle

Table de vérité

A	B	true	false	$\neg A$	$A \vee B$	$A \wedge B$	$A \rightarrow B$	$A \leftrightarrow B$
0	0	1	0	1	0	0	1	1
0	1	1	0	1	1	0	1	0
1	1	1	0	0	1	1	1	1
1	0	1	0	0	1	0	0	0

Calcul propositionnel

$A \wedge \neg A \equiv \text{false},$
 $A \vee \text{true} \equiv \text{true},$
 $A \wedge \text{true} \equiv A,$
 $A \vee \text{false} \equiv A,$
 $A \wedge \text{false} \equiv \text{false},$
 $A \wedge A \equiv A,$
 $A \vee A \equiv A,$
 $A \wedge B \equiv B \wedge A,$
 $A \vee B \equiv B \vee A,$
 $(A \vee B) \vee C \equiv A \vee (B \vee C),$
 $(A \wedge B) \wedge C \equiv A \wedge (B \wedge C),$
 $(A \wedge B) \vee C \equiv (A \vee C) \wedge (B \vee C)$
 $(A \vee B) \wedge C \equiv (A \wedge C) \vee (B \wedge C)$
 $\neg(A \vee B) \equiv \neg A \wedge \neg B,$
 $\neg(A \wedge B) \equiv \neg A \vee \neg B,$
 $\neg\neg A \equiv A$
 $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$
 $A \rightarrow B \equiv \neg A \vee B$

Règles de la déduction naturelle

$$\frac{}{U \vdash A \vee \neg A} \text{ (3Exc)}$$

$$\frac{}{U \cup \{A\} \vdash A} \text{ (Aff)}$$

$$\frac{U \vdash A \rightarrow B \quad U \vdash \neg B}{U \vdash \neg A} \text{ (ABS)}$$

$$\frac{U \vdash A \quad U \vdash A \rightarrow B}{U \vdash B} \text{ (MP)}$$

$$\frac{U \cup \{A\} \vdash B}{U \vdash A \rightarrow B} \text{ (DED)}$$

$$\frac{U \vdash A \vee B \quad U \vdash A \rightarrow C \quad U \vdash B \rightarrow C}{U \vdash C} \text{ (}\vee\text{-Elim)}$$

$$\frac{U \vdash A}{U \vdash A \vee B} \text{ (}\vee\text{-Intro)}$$

$$\frac{U \vdash A \quad U \vdash B}{U \vdash A \wedge B} \text{ (}\wedge\text{-Intro)}$$

$$\frac{U \vdash A \wedge B}{U \vdash A} \text{ (}\wedge\text{-Elim)}$$

$$\frac{U \vdash \neg\neg A \quad U \vdash A}{U \vdash \neg\neg A} \text{ (DN)}$$

$$\frac{U \vdash \neg(A \vee B) \quad U \vdash \neg A \wedge \neg B}{U \vdash \neg(A \vee B)} \text{ (}\vee\text{-De Morgan)}$$

$$\frac{U \vdash \neg(A \wedge B) \quad U \vdash \neg A \vee \neg B}{U \vdash \neg(A \wedge B)} \text{ (}\wedge\text{-De Morgan)}$$

$$\frac{U \vdash \neg A \vee B \quad U \vdash A \rightarrow B}{U \vdash A \rightarrow B} \text{ (}\rightarrow\text{-Déf)}$$